

1982

# Effect of torsion on strength of ship hulls, November 1982 (MA-RD-940-81076)

Alexis Ostapenko

Yaofeng Chen

Follow this and additional works at: <http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports>

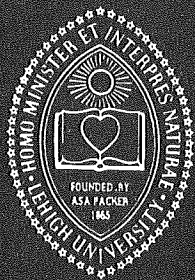
---

## Recommended Citation

Ostapenko, Alexis and Chen, Yaofeng, "Effect of torsion on strength of ship hulls, November 1982 (MA-RD-940-81076)" (1982).  
*Fritz Laboratory Reports*. Paper 518.  
<http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports/518>

This Technical Report is brought to you for free and open access by the Civil and Environmental Engineering at Lehigh Preserve. It has been accepted for inclusion in Fritz Laboratory Reports by an authorized administrator of Lehigh Preserve. For more information, please contact [preserve@lehigh.edu](mailto:preserve@lehigh.edu).

# Lehigh University



## Fritz Engineering Laboratory

LEHIGH UNIVERSITY LIBRARIES



3 9151 00942861 2

*FL Libr*

### MAXIMUM STRENGTH OF SHIP HULLS

### EFFECT OF TORSION ON STRENGTH OF SHIP HULLS

FRITZ ENGINEERING  
LABORATORY LIBRARY

BY  
ALEXIS OSTAPENKO  
YAOFENG CHEN

NOVEMBER 1982

REPORT NO. 468.11

Maritime Administration University Research Program

Report No. MA-RD-940-82076

EFFECT OF TORSION ON  
STRENGTH OF SHIP HULL GIRDERS

Prepared by:

Alexis Ostapenko  
Yaofeng Chen

LEHIGH UNIVERSITY  
Fritz Engineering Laboratory

F.E.L. Report No. 468.11

November 1982

U. S. DEPARTMENT OF TRANSPORTATION, Maritime Administration  
Office of Research and Development

#### LEGAL NOTICE

This report was prepared as an account of government-sponsored work. Neither the United States, nor the Maritime Administration, nor any person acting on behalf of the Maritime Administration:

- (A) make any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- (B) assume any liabilities with respect to the use of or for damage resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "persons acting on behalf of the Maritime Administration" includes any employee or contractor of the Maritime Administration to the extent that such employee or contractor of the Maritime Administration prepares, handles, or distributes, or provides access to, any information pursuant to his employment or a contract with the Maritime Administration.

<b>REPORT DOCUMENTATION PAGE</b>		1. REPORT NO. MA-RD-940-81076	2.	3. Recipient's Accession No.
4. Title and Subtitle EFFECT OF TORSION ON STRENGTH OF SHIP HULL GIRDERS			5. Report Date November 1982	
7. Author(s) Alexis Ostapenko and Yaofeng Chen			8. Performing Organization Rept. No. F.F.L. Rept No. 468.11	
9. Performing Organization Name and Address Lehigh University Dept. of Civil Engineering Fritz Engineering Laboratory #13 Bethlehem, PA 18015			10. Project/Task/Work Unit No. 468	
			11. Contract(C) or Grant(G) No. (C) MA-80-SAC-01081 (G)	
12. Sponsoring Organization Name and Address Maritime Administration U.S. Department of Transportation Washington, D.C. 20590			13. Type of Report & Period Covered FINAL	
15. Supplementary Notes Research was conducted under Maritime Administration 1980 University Research Program			14.	
16. Abstract (Limit: 200 words) A method for the determination of the behavior and maximum strength of longitudinally stiffened single-cell ship hull girders under the combined loading of moment, shear and torque is described. The main features are: (1) Individual components are analyzed under the requirement of deformational compatibility among them; (2) The compression flange is assumed to be made up of parallel beam-columns with the axial force for each defined as a function of the axial deformation (buckling, plastification and large deformations are taken into account); (3) A multiple tension-field action is considered for the webs in the post-buckling stage; (4) Warping of the cross section is incorporated by assuming linear variation of the average axial deformations across the width of each component (flange or web) of the cross section, but without requiring that the cross section remain plane. A previously developed computer program was modified to carry out the analysis, with some data obtained in experiments used as input. This method of analysis shows a better correlation with the experimental results than the method used previously, although the prediction is still somewhat optimistic with respect to the available test results. The following topics are recommended for future investigation: (1) Inclusion of the effect of shear lag on the behavior of flanges; (2) Inclusion of the effect of shearing stress on the behavior of the plate components in the compression flange; (3) Formulation of an energy criterion for determining the optimum degree of warping.				
17. Document Analysis a. Descriptors Hulls (Structures) Ship Hulls Ship Structural Components Research Model Tests Metals b. Identifiers/Open-Ended Terms Ultimate Strength Method Plates Plate Girders Compatibility Methods Stability Methods Longitudinal Stability Buckling Torsion c. COSATI Field/Group 13/10, 13/13, 14/02, 14/06				
18. Availability Statement Approved for Release National Technical Information Springfield, VA 22151		19. Security Class (This Report) UNCLASSIFIED		21. No. of Pages 74
		20. Security Class (This Page) UNCLASSIFIED		22. Price

<b>REPORT DOCUMENTATION PAGE</b>	<b>1. REPORT NO.</b> MA-RD-940-81076	<b>2.</b>	<b>3. Recipient's Accession No.</b>
<b>4. Title and Subtitle</b> EFFECT OF TORSION ON STRENGTH OF SHIP HULL GIRDERS		<b>5. Report Date</b> November 1982	
<b>7. Author(s)</b> Alexis Ostapenko and Yaofeng Chen		<b>8. Performing Organization Rept. No.</b> F.F.L. Rept No. 468.11	
<b>9. Performing Organization Name and Address</b> Lehigh University Dept. of Civil Engineering Fritz Engineering Laboratory #13 Bethlehem, PA 18015		<b>10. Project/Task/Work Unit No.</b> 468	
		<b>11. Contract(G) or Grant(G) No.</b> (C) MA-80-SAC-01081 (G)	
<b>12. Sponsoring Organization Name and Address</b> Maritime Administration U.S. Department of Transportation Washington, D.C. 20590		<b>13. Type of Report &amp; Period Covered</b> FINAL	
<b>15. Supplementary Notes</b> Research was conducted under Maritime Administration 1980 University Research Program		<b>14.</b>	
<b>16. Abstract (Limit: 200 words)</b> A method for the determination of the behavior and maximum strength of longitudinally stiffened single-cell ship hull girders under the combined loading of moment, shear and torque is described. The main features are: (1) Individual components are analyzed under the requirement of deformational compatibility among them; (2) The compression flange is assumed to be made up of parallel beam-columns with the axial force for each defined as a function of the axial deformation (buckling, plastification and large deformations are taken into account); (3) A multiple tension-field action is considered for the webs in the post-buckling stage; (4) Warping of the cross section is incorporated by assuming linear variation of the average axial deformations across the width of each component (flange or web) of the cross section, but without requiring that the cross section remain plane. A previously developed computer program was modified to carry out the analysis, with some data obtained in experiments used as input. This method of analysis shows a better correlation with the experimental results than the method used previously, although the prediction is still somewhat optimistic with respect to the available test results. The following topics are recommended for future investigation: (1) Inclusion of the effect of shear lag on the behavior of flanges; (2) Inclusion of the effect of shearing stress on the behavior of the plate components in the compression flange; (3) Formulation of an energy criterion for determining the optimum degree of warping.			
<b>17. Document Analysis a. Descriptors</b> Hulls (Structures)                      Research Ship Hulls                                  Model Tests Ship Structural Components              Metals  <b>b. Identifiers/Open-Ended Terms</b> Ultimate Strength Method              Stability Methods Plates                                      Longitudinal Stability Plate Girders                              Buckling Compatibility Methods                    Torsion  <b>c. COSATI Field/Group</b> 13/10, 13/13, 14/02, 14/06			
<b>18. Availability Statement</b> Approved for Release National Technical Information Springfield, VA 22151		<b>19. Security Class (This Report)</b> UNCLASSIFIED	<b>21. No. of Pages</b> 74
		<b>20. Security Class (This Page)</b> UNCLASSIFIED	<b>22. Price</b>

## 1. INTRODUCTION

### 1.1 Background

In the conventional methods of ship design, the strength of a ship hull is based on the linear elastic response of the hull components. The section modulus plays an important role in computing the ship hull bending strength. However, more often than not the strength of a ship hull implied by the safety factor is not an accurate indicator of the true ultimate strength. With the growing knowledge of wave loading and the introduction of novel ship types (large tankers, container and special purpose ships), the need for a more realistic evaluation of ship ultimate strength is becoming more important.

Much research has been done on the ultimate strength of individual ship hull components; plate [9, 11, 14], stiffened plate and grillages [4, 6, 10, 20, 23], and plate girders [1, 2, 3, 8, 13, 15, 16, 21, 24, 26]. Caldwell proposed a method of considering the fully plastified cross section in computing the bending moment, for which the post buckling plate strength would be determined by using an effective width at the maximum plate capacity [5]. In the method developed previously in the current research for computing the ultimate strength of ship hull girder under moment, shear and torque, the compression flange was treated as a series of identical beam-columns having the same axial deformation. The

buckling and the post-buckling tension field action of the webs were considered [17]. The method was found to be reasonably accurate for moment and shear loading but over-optimistic for moment, shear and torque. The method presented here represents a further development of that method.

## 1.2 Purpose and Scope

The main purpose of this research was to continue the development of the method for determining the ultimate strength of ship hull girders under the general loading of bending moment, shear and torque. Treatment of the individual components of the ship hull girder is the same as in the previous formulation. The ship hull girder is modeled by a box girder stiffened both in the longitudinal and transverse directions. A typical cross section is shown in Fig. 1 where the transverse stiffeners represent transverse frames or bulkheads in the ship hull and the longitudinally stiffened webs and flanges represent the sides, the deck and the bottom plating.

A fixed relationship between the cross-sectional forces (moment, shear and torque) was assumed for the hull segment to exist at all levels of loading. In analyzing the individual components of the ship hull, the methods derived in prior research were applied to determine the behavior and the ultimate strength of ship bottom plating and side



plating. For the plating of the compression flange, a previously developed computer program was adopted to obtain the axial load vs. average axial deformation relationship [22]. The following principal improvements were made in the method of analysis of the hull girder: (1) Inclusion of the effect of warping, by considering linearly varying axial deformation across the width of the flanges; (2) An equilibrium formulation in which both, the axial force and the bending moment about the vertical centroidal axis, should be equal to zero.

A test specimen from the previous research was analyzed by this method and the computed output was compared with the test results. The comparison indicates the direction for future research to more accurately predict the behavior of ship hull girders.

## 2. THEORETICAL ANALYSIS

### 2.1 Elastic Analysis

#### 2.1.1 Introduction

Although elastic analysis is not applicable for computing the ultimate and post-ultimate behavior, it is suitable for describing the behavior of a hull girder under external loading up to the initiation of non-linear effects due to buckling, yielding or second-order deformations. Thus, linearly elastic analysis of a hull girder is valid under the following conditions:

1. The material properties of all components are linearly elastic.
2. The member is straight and prismatic with the distortion of the cross section being negligible.
3. Residual stresses are small.

#### 2.1.2 Flexural Stresses

Flexural stresses in the box girder cross section can be closely approximated by the simple beam theory :

$$\sigma_b = \frac{M_x y}{I_x} + \frac{M_y x}{I_y} \quad (2.1)$$

where moments  $M_x$ ,  $M_y$ , and  $I_x$ ,  $I_y$  refer to the principal centroidal axes  $x$  and  $y$ .

### 2.1.3 Shearing Stresses

For single-cell thin-walled closed sections loaded in the plane of symmetry the shearing stress is given by:

$$\tau_b = \frac{q_{open}}{t} + \frac{q_o}{t} \quad (2.2)$$

where  $t$  is the wall thickness,  $q_{open}$  is the shear flow for the open section made by introducing an arbitrary cut in the closed section (usually at the vertical axis of symmetry for a symmetrical section),  $q_o$  is the additional shear flow in the plate required to maintain no relative displacement between the two edges of the cut. For a homogeneous section,  $q_o$  is given by:

$$q_o = \frac{\int (q_{open}/t) ds}{\int (ds/t) ds} \quad (2.3)$$

where  $s$  is measured from one end of the cut to the other [7]. Note that longitudinal stiffeners on the webs and flanges enter into the computation of  $q_{open}$ , but only the plate components are involved in the computation of  $\int (q_{open}/t) ds$  and  $\int (ds/t) ds$ .

### 2.1.4 Torsional Stresses

Torsional moment is carried by the cross section in two parts, pure torsion (St. Venant torsion) and warping torsion

$$T = T_{sv} + T_w \quad (2.4)$$

For a general loading case, stresses caused by torque occur in addition to the stresses caused by moment and shear. Uniform shear stress results from the presence of pure torsion and is called St. Venant torsional stress. Warping stresses occur in addition to the St. Venant torsional stress in members of general cross section under torsional loading. Warping shear stress and warping normal stress develop when the section is restrained from deformation.

The differential equation for the deformation of a member subjected to a concentrated torque is:

$$T = GJ(d\phi/dz) - EI_w(d^3\phi/dz^3) \quad (2.5)$$

where  $G$ : shear modulus

$E$ : modulus of elasticity

$\phi$ : angle of twist

$z$ : the coordinate along the longitudinal axis

$I_w$ : warping moment of inertia

The solution for the angle of twist, expressed as a function of  $z$ , is

$$\phi = C_1 + C_2 \cos \lambda z + C_3 \sinh \lambda z + \frac{T}{\lambda^2 EI_w} z \quad (2.6)$$

where  $\lambda = GJ/EI_w$  and the constants of integration depend on the boundary conditions of the beam. Once  $\phi$  is determined, St. Venant shear, warping shear stress and warping normal stress can be readily calculated at any location in the beam. They are:

$$\tau_{sv} = \frac{GJ(d\phi/dz)}{2A_0 t} \quad (2.7)$$

$$\tau_w = - \frac{ES_w(d^3\phi/dz^3)}{t} \quad (2.8)$$

$$\sigma_w = Ew_n(d^2\phi/dz^2) \quad (2.9)$$

where  $A_0$  = area bounded by the box cross section

$S_w$  = warping static moment

$w_n$  = normalized unit warping

The distribution of the St. Venant shear flow, defined by  $\tau_{sv}$ , is constant across the cross section. Figures 2 and 3 show the distribution of  $w_n$  and  $S_w$  in the cross section of a typical ship hull.  $w_n$  and  $S_w$  are functions only of the geometry of the cross section and they directly influence the warping shear stress  $\tau_w$  and the warping normal stress  $\sigma_w$ . The distribution of warping shear stress and warping normal stress is shown in Figure 4 as a function of location across a span, with both ends restrained against warping and subjected to constant torque.

In an actual structure, there is neither free nor fully restrained condition against warping, but the structures are restrained partially. Thus, the actual distribution of stresses caused by torque in a structure can be obtained only approximately, the accuracy being dependent on the degree of torsional restraint at the ends of the segment.

Figures 5 through 7 show the shearing stresses at the mid-length of a typical box girder with longitudinal stiffeners, when the ends are fully restrained against deplanation under the combined effects of moment, shear and torque. Figures 8 through 10 show the normal stress distributions in the same box girder at a cross section near the end.

## 2.2 Non-Linear Behavior

With the assumptions stated in Art. 2.1.1, a ship hull girder has linearly elastic behavior only until a component buckles or starts non-linear behavior. After this, the principle of superposition cannot be applied.

In the proposed method of analysis the following analytical assumptions were used for individual components of the box girder:

1. The effect of shear on the behavior of the compression flange is negligible.
2. Shear stress distribution is uniform in the individual web

subpanels.

3. After a web subpanel has buckled, it can no longer carry any additional normal stress.

The behavior of the hull girder is analyzed by considering the behavior of individual components and enforcing compatibility at the junctions between respective components. The compression flange is under axial loading with or without the presence of lateral load. Non-linearity of the compression flange arises from its non-symmetrical nature; the longitudinal stiffeners are located on one side of the compression flange. The behavior of the compression flange can be modeled by parallel beam-columns hinged or fixed at the transverses. In so doing, the large deformations and strain reversal that take place in the compression flange can be taken into account. In reality, there can either be lateral loading or no lateral load acting on the compression flange, depending on whether the ship deck plating or bottom plating is analyzed. At the present stage the compression flange is assumed to have no lateral load

For the hull girder webs, shear and bending stresses can be computed by simple beam theory as long as the web plate is flat and there are no significant residual stresses. After buckling, the capacity of the buckled subpanel to carry additional normal stress is greatly reduced and a stress redistribution takes place. The

additional normal stress caused by bending moment is then carried by the flanges and the yet unbuckled subpanels. However, the buckled web subpanel can still carry additional shear by the tension-field action. The web subpanel is assumed not to carry any more loading when the shear deformation reaches the point where the diagonal fiber in the subpanel yields.

## 2.3 Behavior of Compression Flange

### 2.3.1 Introduction

The compression flange (deck or bottom plating) of a typical ship hull girder is composed of a plate and a number of longitudinal stiffeners. Behavior of such stiffened plate under axial compression has been studied in the pre- and post-buckling ranges, including the ultimate and post-ultimate ranges [12, 18, 25] and a method for performing analysis has been formed.

In the method presented here, the axial deformation in the compression flange is assumed to vary linearly across the width. Thus, each longitudinal stiffener has a different axial deformation, and the compression flange is treated as if it consisted of a series of individual stiffeners behaving independently from each other. Then, each stiffener with its tributary portion of the plate is treated as a beam-column.



### 2.3.2 Beam-Column

A typical beam-column used to model the response of a particular longitudinal stiffener of the compression flange is shown in Fig. 11. It is subjected to an axial load  $P$ , end moments  $M$  and lateral line loading  $q$ .

The behavior of individual components of the beam-column is treated differently. The overall stress-strain relationship of the plate is described by the average stress vs. overall shortening of the plate (Fig. 12). It takes into account the effect of buckling and residual stresses. The stress-strain relationship of the stiffener is defined by the material property. Most commonly, the material is taken to be linearly elastic-plastic (Fig. 12).

A computer program developed in prior research was used to analyze the beam-column [22]. This method was developed for analyzing ship bottom plating subjected to axial and lateral loads which exist when the ship hull is bent under the hogging moment (Fig. 13). However, when the ship hull is sagging, there is no lateral loading present on the deck plating (Fig. 14) and the computer program cannot be used directly and a modification had to be made. Solutions

for two to three different lateral loading intensities were extrapolated to get the axial load-deformation behavior of the beam-column with zero lateral loading. The resultant response of the beam-column consists of the following three ranges (Fig. 15):

1. Prebuckling: up to  $0.6 - 0.8 P_u$ , the response appears to be linear with deformation mainly due to the axial shortening of the beam column.
2. Non-linear post-buckling flattening till the ultimate load  $P_u$  is reached.
3. Post-ultimate reduction of the load capacity.

### 2.3.3 Effect of Strain Reversal

Special consideration had to be taken for the response of the beam-column in the post-ultimate range. The need for this arose from the direct equilibrium formulation of the method used in which past deformations are not considered. The result of this is that in the post-ultimate range the load-deformation response is unrealistically distorted (Fig. 16) since each point is computed individually without recognizing the fact that the structure may have been subjected to a higher axial load and thus, strain reversal may have taken place.

To remedy this distorted response in the post-ultimate range, the following correction was made to account for strain reversal; the deformation corresponding to the computed axial load was approximated

by assuming;

$$\Delta = \Delta_{pu} + \Delta_c \quad (2.10)$$

where  $\Delta_{pu}$  is the axial shortening at the ultimate load  $P_u$ . The curvature shortening  $\Delta_c$  is kept the same. The behavior computed this way showed reasonable agreement with some test data. A more detailed description of this effect can be found in Ref. [17].

## 2.4 Stresses in Web

### 2.4.1 Prebuckling and Buckling Behavior

For closed box sections under moment, shear and torque, the stresses in the webs are the superimposed effects of moment, shear and torque. Figures 7 and 10 show the normal and shearing stress distributions. The buckling of the web plate or of the web subpanel occurs when the bending, shearing and normal stresses satisfy the following interaction expression:

$$\left(\frac{\tau_{cri}}{F_{vcri}}\right)^2 + \left(\frac{\sigma_{bcri}}{F_{bcri}}\right)^2 + \left(\frac{\sigma_{ccri}}{F_{ccri}}\right) = 1.0 \quad (2.11)$$

where  $\tau_{cri}$  is the shear stress,  $\sigma_{bcri}$  is the bending stress and  $\sigma_{ccri}$  is the compression stress existing in the subpanel, and  $F_{vcri}$ ,  $F_{bcri}$ ,  $F_{ccri}$  are the buckling stresses of a plate panel under shearing, compression or bending stresses, each acting alone. The formulas

defining these critical buckling stresses are listed in Table 1. The plate panels are conservatively assumed to be simply supported at all four edges.

#### 2.4.2 Post-Buckling Behavior

In the post-buckling range, the strength of an individual web subpanel was assumed to develop independently from other subpanels. Thus, the shear force in a web is the sum of the shear capacities of the individual subpanels.

$$V_{web} = \sum V_{subpanels} \quad (2.12)$$

The subpanels can either be in the elastic, buckling or post-buckling state depending on the stress level. After a subpanel has buckled, the normal stress in it is assumed to remain constant and therefore not to contribute to the moment carrying capacity of the box section. The additional shear capacity is computed according to the tension-field action [1, 2, 15, 19]. The ultimate shear strength of the  $i$ -th web subpanel is then the sum of the shear stress at buckling and the tension field action

$$\tau_{ui} = \tau_{cri} + \tau_{tfi} \quad (2.13)$$

where  $\tau_{ui}$  and  $\tau_{tfi}$  are the ultimate shear strength and the

tension-field action contributions to shear in the  $i$ -th subpanel, respectively. The ultimate shear deformation of the  $i$ -th subpanel is assumed to be reached when the diagonal fiber reaches the yield strain [15]. Thus,

$$r_{ui} = \frac{F}{E} \left( \alpha_i + \frac{1}{\alpha_i} \right) \quad (2.14)$$

where  $r_{ui}$  is the ultimate shearing strain, and

$$\alpha_i = \frac{a}{d_i} \text{ is the aspect ratio of the given subpanel.}$$

To simplify the problem, the shear stress-strain relation between the buckling and ultimate levels of stress for each subpanel is assumed to be a straight line. As a result, the shear stress-strain relationship for each subpanel is tri-linear defined by the points at zero, buckling and ultimate stresses. With the assumed shear stress-strain relationship, one has to keep in mind that,

1. After buckling of the subpanel the normal stress in a subpanel remains at the buckling stress level, and any additional normal stresses from the increase in the external load must be distributed to the yet unbuckled subpanels, longitudinal stiffeners and the flanges.
2. After the ultimate shear strain in a subpanel has been reached, the shear stress remains constant as the shear strain increases.
3. When the width-thickness ratio of a particular web subpanel is sufficiently small, the subpanel will yield rather than buckle. The maximum octahedral shearing stress yield criterion (Von Mises) was used to check the yielding condition.

$$3\left(\frac{\tau_i}{F_y}\right)^2 + \left(\frac{\sigma_{ci}}{F_y}\right)^2 \leq 1 \quad (2.15)$$

where  $\tau_i$  and  $\sigma_{ci}$  are the shearing and normal stresses respectively.

Since the shearing stresses in the two webs are different when the box section is subjected to torque in addition to shear and bending, the sequence of buckling for the web subpanels has to be followed separately in each web.

### 3. EFFECT OF WARPING

#### 3.1 Introduction

When a simply supported beam is subjected only to moment and shear, normal stresses in the linearly elastic range can be computed by using Eq. (2.1). However, for a general case, when the torque is also applied, the additional torsional stresses must be considered. These stresses are produced by both, the uniform (St. Venant) and the non-uniform (Warping) torsional actions as discussed in Article 2.1.

The normal stress caused by the bending moment is uniform in the top and bottom flanges of the cross section, whereas torsional actions produce a self-equilibrated and varying normal stress distribution in the cross section. The fact that one is uniform and the other varying provides clues as to whether warping was playing a significant role before any buckling occurs. An analysis was performed on the test segment of the specimen to see if the effect of warping was significant even at small levels of loading.

#### 3.2 Elastic Analysis of Warping

Warping torsion results in normal and shear stresses, and their distributions are similar to those shown in Figs. 2 and 3 for a hull girder cross section as can be seen from Eq. (2.8) and Eq. (2.9). Warping stresses also vary along the segment depending on the

restraint to warping deformations at the ends. Figure 4 shows the distribution of warping normal and shear stresses along a prismatic beam with the warping restrained at the ends. The final state of stress in the box girder is the superimposed effect of the flexural and warping stresses typified by a cross section of the test segment as shown in Figures 7 and 10.

An elastic analysis was performed on the test segment with various extreme boundary conditions assumed to exist at the ends. This was necessary since, for testing one particular segment, the other two segments were reinforced with temporary stiffeners and thus provided some unknown amount of restraint. Also, the end portions of the specimen were composed of thicker plates than in the test segments (Art. 6.1). The two extreme conditions assumed were: both ends prevented from warping (torsionally fixed) or one end restrained and the other end free to warp (torsionally simply supported).

In the case where the ends were restrained against warping at the transverse stiffeners, the distribution of warping normal stress was such that there were large normal stresses at the ends and zero warping normal stress at the mid-length of the segment (Fig. 4).

The other case of the boundary condition was for one end of the



test segment, where there were reinforcements attached to the transverse stiffeners to strengthen the already tested segment, to be strong enough to keep the end from deplanation, and for the other end, where only the plate thickness of the segment changed, free warping was allowed. This lead to the maximum warping normal stress at the mid-length of Test 3 of only  $3.0 \times 10^{-5}$  of the magnitude of the normal stress caused by the bending moment. An analysis of the same boundary conditions (one end fixed and the other end free) for the same test set-up with the fixed end being at the location of the transverse frame where the transverse load acted and the free end being one of the support of the test specimen where the two X-rollers were located (Fig. 19), gave even lower warping normal stress ( $10^{-9}$  of the amount of flexural normal stress). All these assumed boundary conditions seem to indicate that there would be negligible normal warping stresses at the mid-length of the test segment. However, this does not explain the observed strain distribution in the test segment which showed significant variation of the normal stresses across the width of the flanges. This behavior indicates that some other explanation of the variation of normal stresses in the flange is needed for a section under torsion, than warping.

Possible explanations are as follows:

1. The distortion of the cross section as the load applied might have been large enough so that the bending moment was acting on

a cross section other than the doubly symmetrical which was assumed. As a result, the normal stress under bending was no longer uniform in the flanges.

2. The initial deformation of the plate panels might have been such that nonlinear inelastic behavior had already occurred during the early stages of loading so that elastic behavior was precluded.

Both these effects can be eliminated or minimized by using a larger test specimen.

#### 4. DESCRIPTION OF THE METHOD OF ANALYSIS

In the earlier research on the strength of ship hull girders, a computer program was developed which used two strains as variables to define the state of stress in the hull girder cross section. Under the assumption that "a plane section remains plane", the condition of equilibrium for the axial force was sufficient to determine the strain in the tension flange for a particular value of the strain in the compression flange. In other words, a uniform strain distribution was assumed to exist in the compression and tension flanges. A comparison of the computed results with the results of the tests showed that the computer program could quite accurately predict the ultimate strength of a hull girder subjected to moment and shear. However, for the case of moment, shear and torque acting simultaneously, predictions of the ultimate strength were over-estimating the observed results by as much as 70% [17].

It was found that, under the loading of moment, shear and torque, the observed strain distribution was not uniform but had significant and non-planar variation over the box cross section. This indicated that warping took place when torque was applied.

In order to accommodate the deformation of the cross section into the computer programs, the variation of strains between the four

corners had to be considered. As a starting point, linearly varying deformations across the width of the individual components of the cross section were assumed. This line of approach created an immediate difficulty since the number of variables changed from two (one strain in the compression flange and one in the tension flange) to four (the strains at the four corners). Yet, there were only two conditions, viz, resultant axial force and the bending moment about the vertical axis of the cross section to be equal to zero. It was thus necessary to make the following modification to the computer program.

In the equilibrium formulation, the strain readings taken in the test at two corners of the cross section are used as additional input into the computer program in order to compensate for the lack of equilibrium conditions. Then, by enforcing the axial force and the bending moment about vertical centroidal axis to be equal to zero through an iterative process, the other two corner strains are computed. The load parameter corresponding to the computed corner strains can be compared with the test load and to other results obtained in the test.

As a result of the assumption that there is a linearly varying strain distribution across the width of individual components, the formulation of the computer program underwent two major changes.

(1) Instead of treating the compression flange as a set of identical beam-columns having the same axial deformation as was assumed in the uniform strain distribution approach, the compression flange is treated as if it were composed of identical beam-columns each with a different deformation according to the linearly varying strain. Then, the respective axial force in each beam-column is determined from the load-deflection relationship.

(2) In the uniform strain distribution approach, buckling of the corresponding web subpanels occurred in both webs at the same time. Whereas this is true for the loading condition of moment and shear only, the analysis is seriously in error for the case of moment, shear and torque when one web is loaded more heavily in shear than the other (Fig. 6). This effect is taken into account in the present approach by checking the buckling of each web subpanel independently as the deformation in each subpanel increases.

Since in this method two strains are used as input, there is no guaranty that the computed  $M_x$  (and the load parameter  $W$ ) corresponds to the true solution of the problem. Actually, the method gives an upper bound solution, that is, the computed  $M_x$  (or  $W$ ) should be greater or, at best, equal to the actual moment  $M_x$ .

The scope of the present study has been mainly concerned with the development of the many computational procedures needed in the programming of the method and, thus, the approach was not checked for a general case, but only for the specimens tested in the previous phase of this research program.

## 5. COMPUTER PROGRAMS

### 5.1 Introduction

Two computer programs were used for analyzing the behavior and ultimate strength of ship hull girders. The first program, developed in previous years, was used to generate the axial load-deformation response of longitudinally stiffened panels with fixed or simply supported end conditions. Two to three axial load-deformation relationships under different lateral loading were first produced and then extrapolated to the condition of zero lateral load.

The extrapolated axial load-deformation response modelled the behavior of the compression flange of the box girder in the second computer program which was used to analyze the load deformation response of a typical ship hull girder segment subjected to a simultaneous action of moment, shear and torque. The values of moment, shear and torque were assumed to remain in the same ratio to each other and thus, for convenience, were related to each other through a load parameter equivalent to a concentrated load acting on a simply supported beam. This load parameter is, for brevity, referred to as the load in this presentation. The output was an array of load (load parameter) values versus the average axial deformation values of the compression flange.

Present effort was mainly concerned with the development of the box girder program; the merging of the two programs into one is recommended for future work.

## 5.2 Computer Program for Stiffened Plate

### 5.2.1 Brief Description

An already developed computer program for analyzing the axial load vs. deformation response of stiffened panels was used [22]. However, some modifications had to be made in order to use it in the present study.

One of the modifications involved the programming of the extrapolation of the output of two or three runs for different small values of lateral loading to the desired output corresponding to zero lateral load. This extrapolation was needed for the case where the compression flange of the box girder represented the deck plating and had no lateral loading. The number of runs used depended on whether linear or parabolic extrapolation was desired.

Detailed development of the original computer program is described elsewhere [10, 22]. The important features of it include the following.

1. The beam-column is composed of a plate and longitudinal



stiffener. The stiffener may be with or without a flange.

2. While an average stress vs. overall strain relationship was assumed for the plate component, the longitudinal stiffener web and flange were assumed to have a linearly elastic-plastic material properties (Fig. 12). The web and flange of the stiffener may have different yield stress values.
3. The effect of residual stresses in the plate can be included.
4. The ends of the beam-column can be fixed or simply-supported.

### 5.2.2 Extrapolation

The output obtained from the stiffened panel computer program was an array of axial loads and deformations for a certain non-zero lateral load. In order to model the response of the compression flange of the box girder when there was no lateral loading, the behavior of the stiffener panel for some two or three small lateral load values was extrapolated to the case of zero lateral load.

Normally, two to three axial load-deformation arrays associated with lateral loading values between  $\bar{q} = 0.03$  to 0.15 produced good extrapolated results.

$$\bar{q} = -\frac{q}{\sigma_o} \frac{r}{A} \quad (5.1)$$

where  $q$ : lateral loading per unit length of beam column

$r$ : radius of gyration of cross section

$\sigma_o$ : buckling stress

A: area of cross section of beam-column

The resultant load-deformation relationship is in the following form (Fig.15):

$$\frac{P}{P_y} = f\left(\frac{\Delta/L}{\epsilon_y}\right) \quad (5.2)$$

Where  $\Delta$  is the axial deformation of the beam-column.

The computational process of extrapolation involved first selecting the array that had the minimum post-ultimate axial deformation. The curve length of the selected array was determined from the linear distances between one data point to the next. Based on this curve length, a set of new points with equal linear distance were established on the curve. The number of these points defined the number of the desired extrapolation data points and their abscissae. With the abscissa of a desired data point known, seven data points from one of the input data arrays were chosen around the neighborhood of the point to be extrapolated, and a cubic curve fitting performed to determine its ordinate.

Following this same procedure for the second and the third array around the selected abscissa value, either a linear or a parabolic

extrapolation was used to find the ordinate, which was the axial force corresponding to a zero lateral loading. This extrapolation was performed point by point until every axial force data point in the extrapolated array was calculated.

The adoption of curve fitting through seven points instead of interpolating between the data points was necessary because the data points computed by the stiffened plate computer program had a certain degree of approximation and, thus, did not result in a smooth curve passing through them. In consequence, when extrapolation was performed through the points interpolated between two neighboring computed points, the extrapolated points (for zero lateral loading) exhibited a very erratic pattern. Some such points even fell below all the data points used for extrapolation, although they were expected to be above. The use of the seven-point curve fitting produced a reasonably smooth extrapolated curve which was very close to the results obtained previously by manual extrapolation [17].

### 5.3 Box Girder Computer Program

#### 5.3.1 Background and Assumptions

The box girder computer program performs the analyses of a typical segment of ship hull girder bounded by two transverse stiffener rings. The segment is of a single-cell box shape composed

of plating and longitudinal stiffeners in the flanges and webs. The input to the computer program includes the hull girder geometry, the response of the stiffened panel under compression as obtained from the computer program described above (Art. 5.2), the moment, shear and torque in terms of their relationship to the load parameter, and the stress-strain relationships of the web plate, web stiffeners and the tension flange. The principal output consists of an array of the concurrent values of the load parameter and of the deformation defined by the average of the axial strains at the two edges of the compression flange. Supplementary optional output gives a detailed picture of the stress and deformation conditions for each load value.

In the method used in the preceding research, it was assumed that "a plane section remains plane" with the consequence that the axial deformation in the flanges was uniform and only two deformation parameters (top and bottom corner strains) were used as variables. In the present program, three deformational parameters are involved, as described in Chapter 4. Among the basic assumptions used in the new computer program are the following:

1. The box girder segment is straight and prismatic, the cross section is single-cell and symmetric about the vertical axis.
2. Effect of shear lag is neglected.
3. Variation of the axial deformation between the four corners is linear.

4. Warping shear is negligible.
5. There are no gross residual stresses in the box girder section, and the effect of residual stresses on the behavior of the compression flange is incorporated into the axial load vs. deformation relationship obtained by a separate computer program.
6. After a web subpanel has buckled, the normal stress is assumed to remain constant at the buckling level.

### 5.3.2 Computational Procedure

The program calculates the load parameter for successive input values of edge strains. The deformation input consists of the edge strains at the two corners of the box girder segment,  $\epsilon_1$  and  $\epsilon_4$  as obtained from Test 3 (fig.21). The strain at the other two corners is interpolated by enforcing equilibrium of the cross section with respect to the axial load and the bending moment about the vertical centroidal axis, both of which are zero in this case.

After the equilibrium of the cross section is achieved, the moment about the horizontal centroidal axis is calculated. This moment gives the load parameter  $W$  and the corresponding shear and torque from the following moment( $M$ ), shear( $V$ ), torque( $T$ ) and load parameter( $W$ ) relationships,

$$M = C_1 W \quad (5.3)$$

$$V = C_2 W \quad (5.4)$$

$$T = C_3 W$$

(5.5)

The shear stresses, which include the effects of flexural shear and St. Venant torsional shear, are distributed to the web subpanels. The interaction of the bending stress, normal stress and shear stress is then checked for each subpanel to see if they have made the subpanel buckle. If no new subpanels have buckled, the calculated corner strains and the load parameter represent one state of response for the hull girder under given loading. If one of the subpanels has buckled, the overall strain is decreased and several cycles of iteration are performed to get the theoretical buckling state before the next set of corner strains is input. After a particular subpanel buckles, its normal stress is kept at the value of buckling. The web subpanel can continue to carry additional shear stress by means of the tension-field action. Repeated strain input produces an array giving the load response of the hull girder from zero load to the ultimate and through the post-ultimate range.

In addition to checking buckling interaction due to bending, normal and shearing stresses, the stress condition was checked against von Mises yield criterion. In the post-ultimate range where the load was decreasing with an increasing strain input, the corresponding decrease in shear was assumed to be linearly elastic.

## 6. DESCRIPTION OF HULL GIRDER TESTS

### 6.1 Test Specimen

To check the soundness of the assumptions made in the analytical method, a test specimen was fabricated and three tests conducted. Figure 17 shows the test specimen and the location of the test segments. In the first test, only moment and shear were applied while torque was added in the second and third tests. The values of moment shear and torque are shown in terms of the load parameter.

Two of the tests (T1 - bending and shear, and T2 - bending, shear and torque) are described in Reference [17]. The third test (T3 - moment, shear and torque) was conducted later and is briefly introduced here. A summary of this test program is outlined below.

The specimen was designed to model a portion of a typical ship hull girder. Figure 18 shows two views and two sections of the test specimen. Its overall dimensions were: length- 2972mm (117in.), width- 667mm (26.25in.), and depth- 508mm (20in.). In the middle portion [1372mm long (54in.)], the plate thickness was 1.85mm(0.073in.). The end portions were 686mm(27in.) and 914mm(36in.) long, and the thickness of the flange plate was 6.35mm(1/4in.) and of the web plate 3.18mm(1/8in.)

Four transverse stiffeners 51mm(2in.) by 9.53mm(3/8in.) divided the mid-portion of the test specimen into three equal segments 457mm(18in.) long. In the test segments, there were five longitudinal stiffeners 3.18mm(1/4in.) by 20mm(0.787in.) spaced evenly on the outside of the compression and tension flanges. On the webs, there were two longitudinal stiffeners of the same size as on the flanges, located at 168mm(6.625in.) from the top and bottom flange plates.

Although in an actual ship hull structure, all stiffeners would be all located on the inside, in the test specimen, the stiffeners were put on the outside. This was done in consideration of the ease of inspection during fabrication and the convenience of observation of the behavior of the stiffeners, flanges and webs during testing.

## 6.2 Test Setup

At one end (left end in Fig. 19), the box girder section was supported on two X-rollers, one roller under each web. The X-rollers made it possible for the specimen to rotate only in the longitudinal direction. At the other end of the specimen (right end in Fig. 19), an X-Y roller was used so that the specimen could pivot and move in both the longitudinal and transverse directions. This support arrangement was designed to make the girder simply supported for bending moment and shear and to carry all the torque in the portion



between the load point and the end supported on the two X-rollers (left end).

The external load was applied by means of a hydraulic jack positioned between the transverse test frame and a spreader beam which was supported on two plates welded to the two vertical transverse stiffeners on the test specimen. Thus, the load was applied to the test segment through the webs. Torque was introduced by positioning the jack with the specified eccentricity with respect to the centroid of the girder.

During the testing of a particular segment, the yet untested segments were protected by reinforcement to prevent undue distortions. The already failed test segments were reinforced in order not to interfere with the testing of the next segment [17].

### 6.3 Loading Conditions

For the three tests conducted, there was constant proportionality between the external load applied at the hydraulic jack and the moment, shear and torque at the mid-span of each test segment. The moment and shear were simply the moment and shear at the mid-length of the test segment due to the load applied to the whole specimen simply supported at the ends (Fig. 17). In Test 1, the load was applied

at the center of the transverse frame so that no torque was produced. The torque in Test 2 and Test 3 was achieved by applying the jack with some prescribed eccentricity.

Constants  $C_1$ ,  $C_2$  and  $C_3$  as defined in Eq.(5.3) through Eq.(5.5) related the moment shear and torque to the load parameter. In this case, the vertical load  $W$  applied to the specimen was used as the load parameter. The theoretical ultimate load, the ultimate load as recorded in the experiment, and their ratios for all the three tests are listed as follows:

<u>Constants and Ultimate Loads</u>			
Test	1	2	3
$M/W = C_1[m]$	0.562	0.616	0.615
$V/W = C_2$	0.615	0.385	0.578
$T/W = C_3[m]$	0	0.194	0.200
Load $W_u$			
$W_{theo}[kN]$	306.9	280.2	276.9
$W_{exp}[kN]$	266.9	164.6	192.9
$W_{theo}/W_{exp}$	1.15	1.70	1.44

The ratios of the theoretical and experimental ultimate loads show that the estimate was good when torque has not introduced (only 15% overestimate), but considerable difference occurred when torque

was applied.

#### 6.4 Analysis of Test 3

As an illustration of the application of the developed computer program, it is used here to analyze Test 3. The corner strain readings at the edges of the compression flange (at the mid-length of the test segment) were used as input to the computer program (Fig. 21).

An examination of the strain distribution in the compression flange of the test segment showed that there was a very large variation (Fig. 20). If only the corner strains at the edges in the compression flange were used and the large variation in-between ignored, the resulting axial force in the compression flange would have appeared to be misleadingly larger than expected in reality.

In an attempt to compensate for this variation of strains in the test specimen, when only the corner strains had to be used in the computer program, a linear fit of the strain readings in the compression flange was performed to give an equivalent linearly varying strain pattern. The relationship between the ratio of the reduced corner strains to those observed for the corresponding load parameter appeared to be nearly constant (Fig. 21):

$$\epsilon_{e1} = 0.68 \epsilon_1 \quad (6.1)$$

$$\epsilon_{e2} = 0.38 \epsilon_2 \quad (6.2)$$

For a particular set of input strains, these two reduction coefficients were applied to the edge strains in the compression flange involved in the computation of forces. As for the other parts of the cross section, they were still subjected to the same unreduced strains. This approach gave the following results:

1. The computed  $\epsilon_3$  were almost twice as large as the  $\epsilon_3$  observed.
2. The computed  $\epsilon_1$  remained within 5% of the observed value for the loads up to the ultimate load. In the post-ultimate range, the computed  $\epsilon_1$  showed a strain reversal that was contrary to the test observation that the strain kept increasing in that range.
3. The computed ultimate load was 233.3kN(52.45kips) as compared to the observed ultimate load of 177.9kN(40kips), which constituted an 30% over-estimate as compared to the 70% over-estimate obtained from the previous computer program.

The fact that the computed  $\epsilon_3$  value was much larger than the observed value can be partially explained by the following:

1. The strain gages measured only the strain on the plate surface rather than the true average, thus, the accuracy of the strain readings may be in question.
2. The fact that the compression flange deflected more at the mid-portion than at the ends made the measured strain in that part less pronounced. Still, this could account only partly for the unusually large variation of the observed strains (Fig. 20). Apparently, the effect of shear lag, which causes the same type

of strain distribution, should be considered to improve the accuracy.

3. Distortion of the cross section, which was not accounted for in the present analysis, could have affected the measured strain values.

## 7. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

### 7.1 Summary

A theoretical analysis was made of the behavior of ship hull girder segments between two transverses. The main purpose was to study the behavior in the pre- and post-ultimate ranges under the combined loading of moment, shear and torque. The results predicted by this analysis were compared to previously conducted test observations to verify the assumptions of the proposed method.

The method adopted in the analysis consisted of considering the behavior of the individual components comprising the ship hull girder through continuous interaction among them from a zero load up to the ultimate and post-ultimate stages of loading. Compatibility was maintained at the junctions between the components as the load increased. The compression flange was modelled by parallel beam-columns, each consisting of a longitudinal stiffener and the tributary portion of the plate. Each beam-column was assumed to have the axial deformation corresponding to a linear variation of the strain across the width of the compression flange. Buckling and post-buckling, plastification and residual stresses were all taken into account. The webs were analyzed by considering the behavior of individual web subpanels in the buckling, post-buckling and ultimate ranges.

A previously developed computer program was considerably modified in order to more properly model the behavior of a ship hull girder under combined loading. The principal modifications were the following:

1. The effect of warping of the ship hull cross section was considered by assuming a linearly varying strain distribution between the junctions of the flanges and the webs. This feature was included as a result of the observations in the previously conducted tests that under combined loading there was significant warping of the cross section.
2. In the modelling of the behavior of the ship hull girder webs, the fact that the individual web subpanels buckle at different stages of loading was recognized. Buckling of the web subpanels could develop independently of each other.

The procedure of the method in the computer program relied on inputting two corner strains. For expediency, the strains obtained from a test were used. Using an average strain value in the compression flange, the two computed corner strains, together with the computed load parameter constituted the results for comparison with the observed results.

This comparison of the analytical and experimental results for Test 3 showed that:

1. Predicted  $\epsilon_1$  strain values (Fig. 21) follow closely the obtained strain up to the ultimate stage.
2. Predicted  $\epsilon_3$  values were as much as twice of the values from test observation.
3. Computed ultimate strength of the test segment was 30% higher than the observed value.

## 7.2 Conclusions

Results of the three tests conducted shows that for the loading case of moment and shear, the prediction by the analytical method comes close to the experimental results but was optimistic (an 15% over-estimate). In the tests where torque was involved (Test 2 and Test 3), the method utilized needs refining as it showed as much as 70% over-estimate.

Considering that essentially arbitrary corner strains have to be input in the present method to participate in the iteration to reach equilibrium, there might be the same external load computed when inputting another set of corner strains. This leads to the conclusion that the calculated external load would be always larger than the value obtained from a test and at best equal to it, i.e., the strains produced by the external loadings in the experiment will always give the lower bound of the potential energy provided by the particular boundary conditions in the test. Any other set of corner strains producing the same external loading will have a potential energy larger than that.



### 7.3 Recommendations

The following recommendations with regard to future research are made:

1. Establishment of an energy criterion is needed to determine the minimum potential energy in the hull girder segment under various combinations of corner strain sets. This criterion will eliminate the present need of inputting two strains to compute the external load parameter.
2. Consideration of one web gradually becoming weaker as the external load increases to the point that the cross section, in effect, becomes an open section (relocation of the shear center).
3. Incorporation of the effect of shear on the behavior of the compression flange plate subpanels.
4. Incorporation of the effect of shear lag into the behavior of individual plate components.
5. Tests on larger specimens so that the effect of geometric imperfections could be minimized.

## 8. NOMENCLATURE

A	Area of cross section of the beam-column
$A_0$	Area bounded by the walls of the box cross section
$C_i$	Integration constants for Eq.(2.6) Loading constants for Eqs.(5.3), (5.4), and (5.5)
E	Modulus of elasticity
$F_{bcri}$	Buckling stress of a plate under bending alone
$F_{ccri}$	Buckling stress of a plate under compression alone
$F_{vcri}$	Buckling stress of a plate under shearing alone
$F_y$	Yield stress
G	Shear modulus
$I_w$	Warping moment of inertia
$I_x, I_y$	Moments of inertia with respect to axes x and y
J	Torsional section constant
M	Moment ( $=M_x$ )
$M_x, M_y$	Bending moment with respect to centroidal axes x and y
P	Axial force
$P_u$	Ultimate axial force
$P_y$	Axial force which causes full yielding of cross section
$S_w$	Warping static moment
T	Torsional moment
$T_{sv}$	Torsional moment due to St. Venant torsion
$T_w$	Torsional moment due to warping torsion
V	Shear force
q	Lateral load per unit length of beam-column
$q_0$	Shear flow required to maintain compatibility of a closed thin

	walled cross section
$q_{open}$	Shear flow in an open thin walled cross section
$r$	Radius of gyration of cross section
$r_{ui}$	Ultimate shear strain of the $i$ -th subpanel
$s$	Coordinate along the wall of open cross section
$t$	Wall thickness of cross section
$w_n$	Normalized unit warping
$x, y, z$	Coordinate axes
	Axial shortening of beam-column
$\Delta_c$	Axial shortening of beam-column due to curvature
$\Delta_{pu}$	Axial shortening of beam-column due to ultimate load $P_u$
$\alpha_i$	Aspect ratio of the $i$ -th web subpanel
$\epsilon_i$	Edge strain at corner $i$
$\epsilon_{ei}$	Reduced edge strain at corner $i$
$\epsilon_y$	Yield strain $=F_y/E$
$\lambda$	Torsional constant ratio $(GJ/EI_w)$
$\sigma_b$	Flexural stress
$\sigma_{bcri}$	Bending stress in $i$ -th subpanel at buckling
$\sigma_{ccri}$	Compression stress in $i$ -th subpanel at buckling
$\sigma_i$	Normal stress in the $i$ -th subpanel at buckling
$\sigma_o$	Column buckling stress
$\sigma_w$	Normal stress due to warping
$\tau_b$	Shear stress due to transverse shear force on closed section
$\tau_{cri}$	Shear stress in the $i$ -th subpanel at buckling
$\tau_i$	Shear stress in the $i$ -th subpanel
$\tau_{sv}$	Shear stress due to St. Venant torsion

468.11

$\tau_{tfi}$	Tension-field action contribution to shear stress in the i-th subpanel
$\tau_{ui}$	Ultimate shear stress of the i-th subpanel
$\tau_w$	Shear stress due to warping torsion
$\phi$	Angle of twist

## REFERENCES

- [1] K. Basler.  
"Strength of Plate Girders Under Combined Bending and Shear,"  
Journal of the Struct. Div., ASCE, Vol. 87, No. ST7, Proc.  
Paper 2968, October 1961.
- [2] K. Basler.  
"Strength of Plate Girders in Shear," Journal of the Struct.  
Div., ASCE, Vol. 87, No. ST7, Proc. Paper 2967, October 1961.
- [3] K. Basler and B. Thurlimann.  
"Strength of Plate Girders in Bending," Journal of the Struct.  
Div., ASCE, Vol. 87, No. ST7, Proc. Paper 2913, August 1961.
- [4] H. Becker and A. Colao.  
"Compressive Strength of Ship Hull Girders," (Part III, Theory  
and Additional Experiments), SSC No. 267, Ship Structure  
Committee, Washington, 1977.
- [5] J. B. Caldwell.  
"Ultimate Longitudinal Strength," Proceedings of the Royal  
Institution of Naval Architects, Vol. 107, London, 1965.
- [6] J. Dwight.  
"Collapse of Steel Compression Panels," Trans. Developments in  
Bridge Design and Construction, University College, Cardiff,  
1971.
- [7] T. V. Galambos.  
"Structural Members and Frames," Englewood Cliffs, N. J.,  
Prentice Hall, 1968.
- [8] M. Herzog.  
"Ultimate Strength of Plate Girders From Tests, Journal of the  
Struc. Div., ASCE, Vol. 100, No. ST5, Proc. Paper 10530, May  
1974.
- [9] W.T.Koiter.  
"The Effective Width of Flat Plates for Various Longitudinal  
Edge Conditions at Loads Far Beyond Buckling Load," National  
Luchtvaartlaboratorium, Netherlands, Rep. S287 (in Dutch),  
1943.
- [10] J. Kondo.  
"Ultimate Strength of Longitudinally Stiffened Plate Panels  
Subjected to Combined Axial and Lateral Loading," Fritz  
Engineering Laboratory Report No. 248.13, Lehigh University,  
1965.
- [11] T.T.Lee.  
"Elastic-Plastic Analysis of Simply Supported Rectangular Plates  
Under Combined Axial and Lateral Loading," Fritz  
Engineering Laboratory Report No. 248.7, Lehigh University,  
1961.
- [12] A. E. Mansour.  
"Charts for the Buckling and Post-Buckling Analysis of Stiffened

Plates under Combined Loading," Technical and Research Bulletin No. 2-22 Society of Naval Architects and Marine Engineers, July, 1976.

- [13] Ch. Massonnet.  
"Design of Steel Plate and Box Girder Bridges," Journal of Struc. Div., ASCE, Vol. 101, No. 11, Proc. Paper 11686, p.2477, 1975.
- [14] K.E. Moxham and C.D. Bradfield.  
"The Strength of Welded Steel Plates Under Inplane Compression," University of Cambridge, Department of Engineering, Technical Report No. CUED/C-Struct/TR.65, 1977.
- [15] A. Ostapenko and C. Chern.  
"Ultimate Strength of Longitudinally Stiffened Plate Girders under Combined Loads", IABSE Proceedings, Vol. 11, II, London, 1971.
- [16] A. Ostapenko, C. Chern and S. Parsanejad.  
" Ultimate Strength Design of Plate Girders", in Developments in Bridge Design and Construction, Crosby, Lockwood and Son, Ltd., University College, Cardiff, 1971.
- [17] A. Ostapenko and A. Vaucher.  
"Ultimate Strength of Ship Hull Girders under Moment, Shear and Torque," MARAD Report No. MA-RD-940- 80077 (Fritz Engineering Laboratory Report No. 453.9), July 1980.
- [18] A. Ostapenko.  
"Ultimate Strength Design of Wide Stiffened Plates Loaded Axially and Normally," in Proceedings of Symposium on Structural Analysis, Non-Linear Behavior and Techniques, Transport and Road Research Laboratory (TTRL) Supplementary Report 164 UC, p. 175, 1975.
- [19] A. Ostapenko.  
"Shear Strength of Longitudinally Stiffened Plate Girders," Proceedings of Structural Stability Research Council, 1980.
- [20] S. Parsanejad and A. Ostapenko.  
"Ultimate Strength Analysis of Plate Grillages under Combined Loads," Fritz Engineering Laboratory Report No. 323.11, Lehigh University, 1972.
- [21] K. C. Rockey, H.R. Evans and D. M. Porter.  
"A Design Method for Predicting the Collapse Behavior of Plate Girders," Proc. Institution of Civil Engineers, Part 2, Vol. 65, p. 85-112, March 1978.
- [22] D. R. Rutledge.  
"Computer Program for Ultimate Strength of Longitudinally Stiffened Panels," Fritz Engineering Laboratory Report No. 248.23, Lehigh University, 1968.
- [23] D. R. Rutledge and A. Ostapenko.  
"Ultimate Strength of Longitudinally Stiffened Plate Panels (Large and Small b/t, General Material Properties) ," Fritz

Engineering Laboratory Report No. 248.24, Lehigh University,  
1968.

- [24] W. Schueller and A. Ostapenko.  
"Tests on a Transversely Stiffened and on a Longitudinally  
Stiffened Unsymmetrical Plate Girder , WRC Bulletin No. 156,  
November 1970.
- [25] C. S. Smith.  
"Compressive Strength of Welded Steel Ship Grillages,"  
Proceedings of the Royal Institution of Naval Architects,  
Vol. 117, London, 1975.
- [26] J. D. Vernarr.  
"Comparison Study of Plate Girder Ultimate Strength Methods,"  
Report CE 481, Department of Civil Engineering, Lehigh  
University, September 1977.

## 10. ACKNOWLEDGMENTS

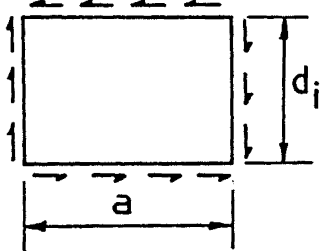


This research was conducted at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. Dr. D. A. VanHorn is Chairman of the Civil Engineering Department and Dr. L. S. Beedle is Director of the Laboratory.

The authors are grateful for the financial support provided by the United States Department of Commerce through the Maritime Administration University Research Program Contract MA-80-SAC-01081. They particularly acknowledge the encouragement and advice given by Mr. Frederick Seibold, the Program Manager of the Office of Maritime Technology, and Dr. Walter M. Maclean, the Contract Officer's Technical Representative (COTR).

Many thanks are due to Mrs. Meng-Ling Chen and Miss Shirley Matlock for processing the text of the report and to Messrs. Peter A. Ostapenko and Adang Surahman for helping with the reduction of test data and other technical tasks.



Table 1 Reference buckling stresses

Pure Stress	Aspect Ratio $\alpha = a/d_i$	Buckling Coefficient $k$	Relative Plate Slenderness	For $\lambda$	Buckling Stress/Yield Stress
<b>Shear</b> 		$k_v = 5 + \frac{5}{\alpha^2}$	$\lambda_v = 0.8 \frac{d_i}{t_w} \sqrt{\frac{F_y}{E k_v}}$	$\leq 0.58$	$\frac{F_{vcr}}{F_y} = 0.58$
				$\geq 0.58$ $\leq 1.41$	$\frac{F_{vcr}}{F_y} = 0.58 - 0.357(\lambda_v - 0.58)^{1.18}$
				$> 1.41$	$\frac{F_{vcr}}{F_y} = 0.58(1/\lambda_v^2)$
<b>Bending</b> 	$> \frac{2}{3}$	$k_b = 24$	$\lambda_b = \frac{d_i/t_w}{0.95} \sqrt{\frac{F_y}{E k_b}}$	$\geq 0.65$ $\leq 1.5$	$\frac{F_{bcr}}{F_y} = 0.072(\lambda_b - 5.62)^2 - 0.78$
	$< \frac{2}{3}$	$k_b = 24 + 73(\frac{2}{3} - \alpha)^2$		$\geq 1.5$	$\frac{F_{bcr}}{F_y} = 1/\lambda_b^2$
<b>Axial</b> 	$> 1$	$k_c = 4$	$\lambda_c = \frac{d_i/t_w}{0.95} \sqrt{\frac{F_y}{E k_c}}$	$\geq 0.65$ $\leq 1.5$	$\frac{F_{ccr}}{F_y} = 0.072(\lambda_c - 5.62)^2 - 0.78$
	$< 1$	$k_c = (\alpha + \frac{1}{\alpha})^2$		$\geq 1.5$	$\frac{F_{ccr}}{F_y} = 1/\lambda_c^2$

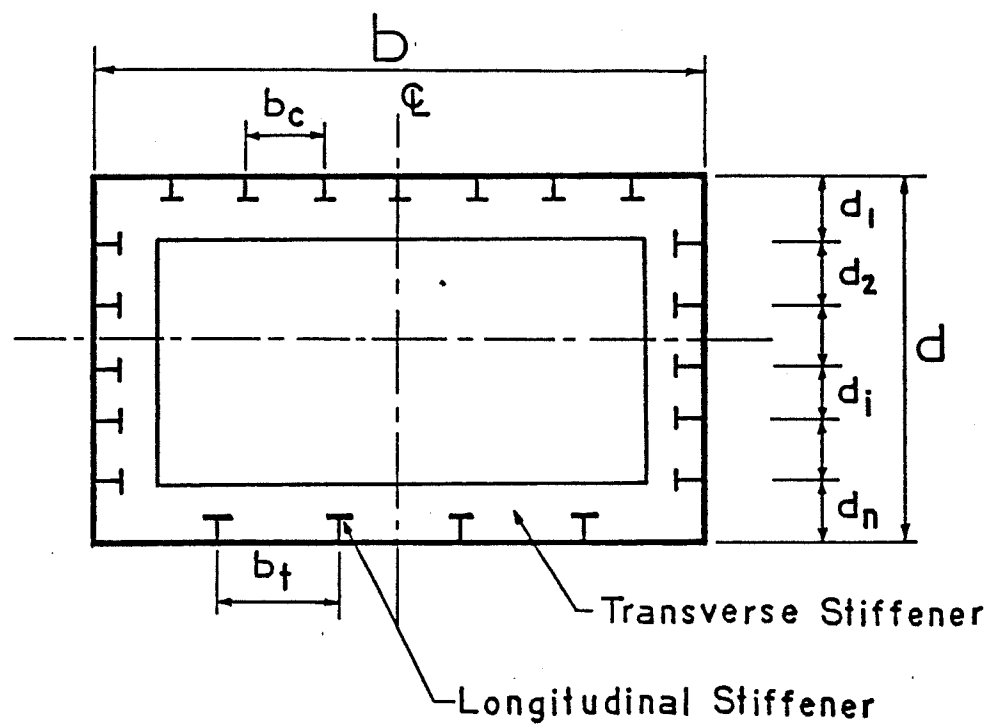


Fig. 1 Idealized ship cross section

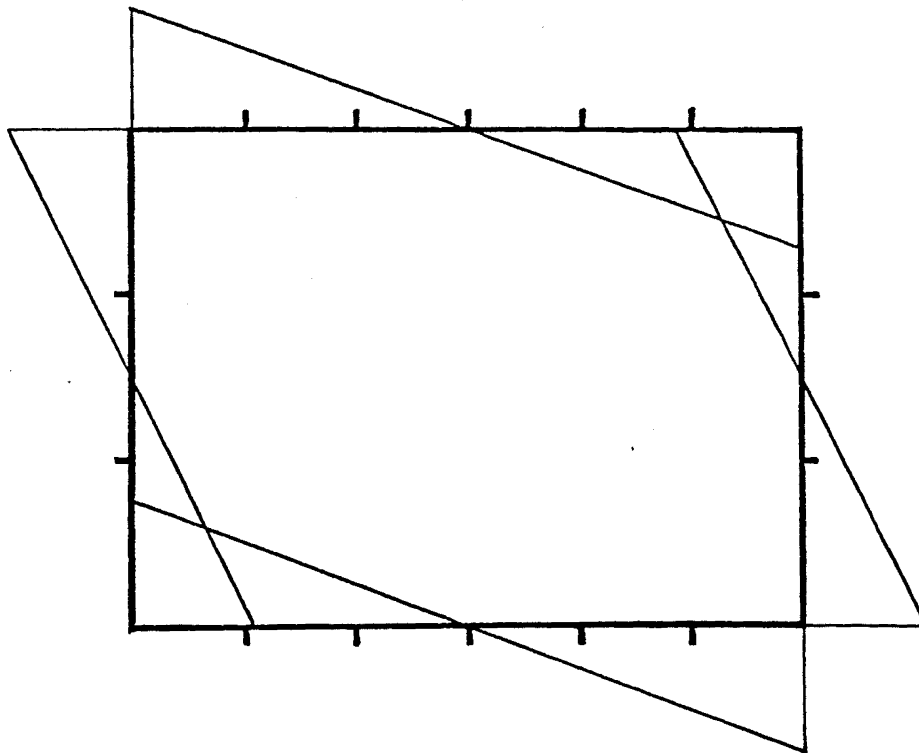


Fig. 2 Distribution of the normalized unit warping  
in the cross section

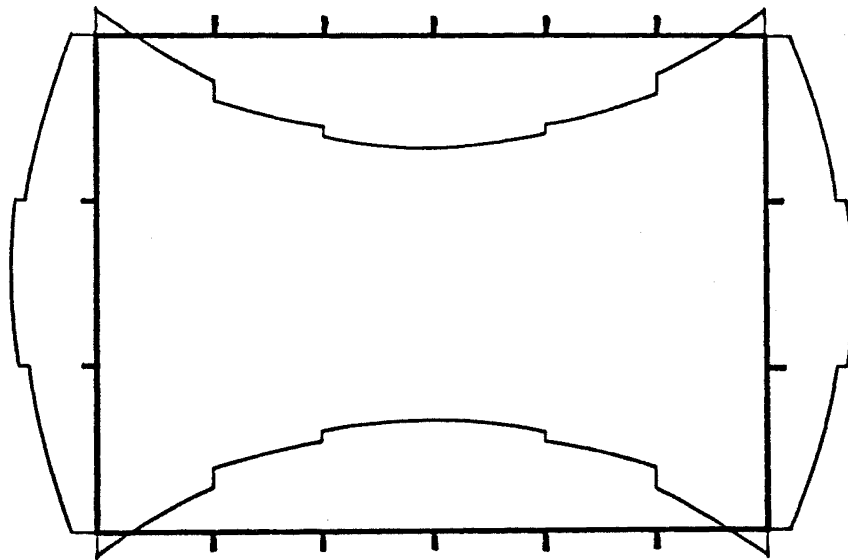


Fig. 3 Distribution of the warping statical moment  
in the cross section

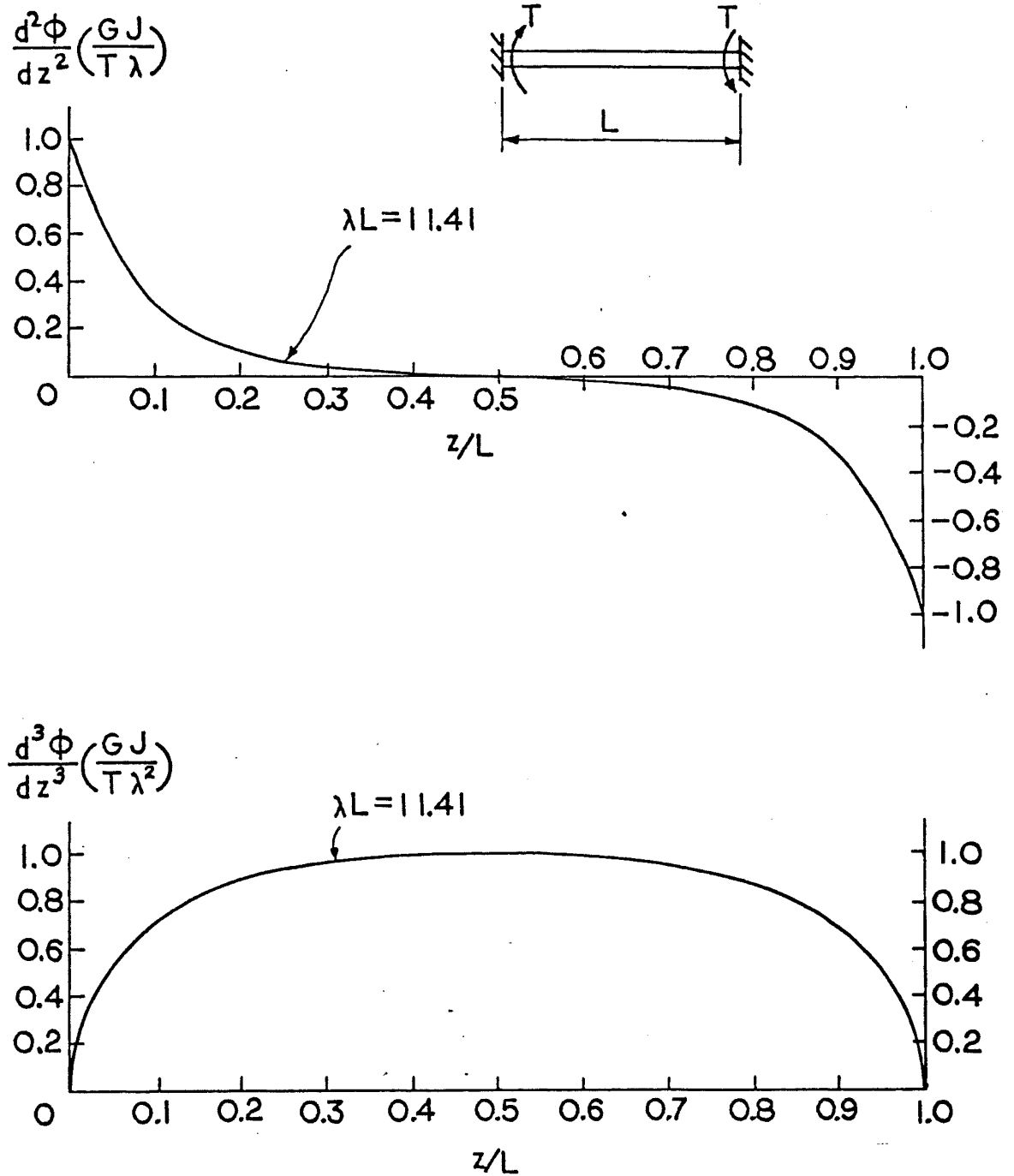


Fig. 4 Distribution of warping normal stress and warping shear stress along a span subjected to constant torque with warping deformation restrained

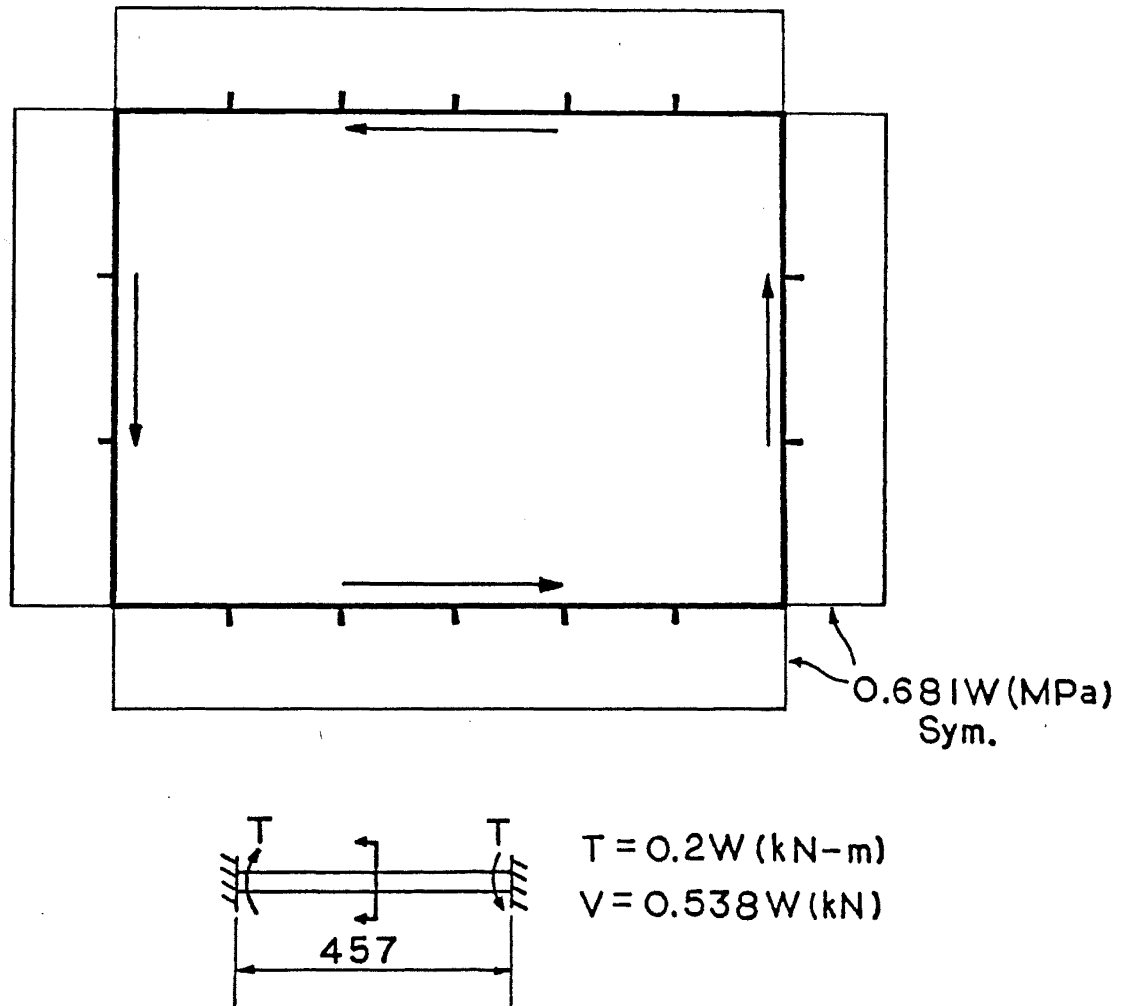


Fig. 5 Distribution of St.Venant shear in the cross section

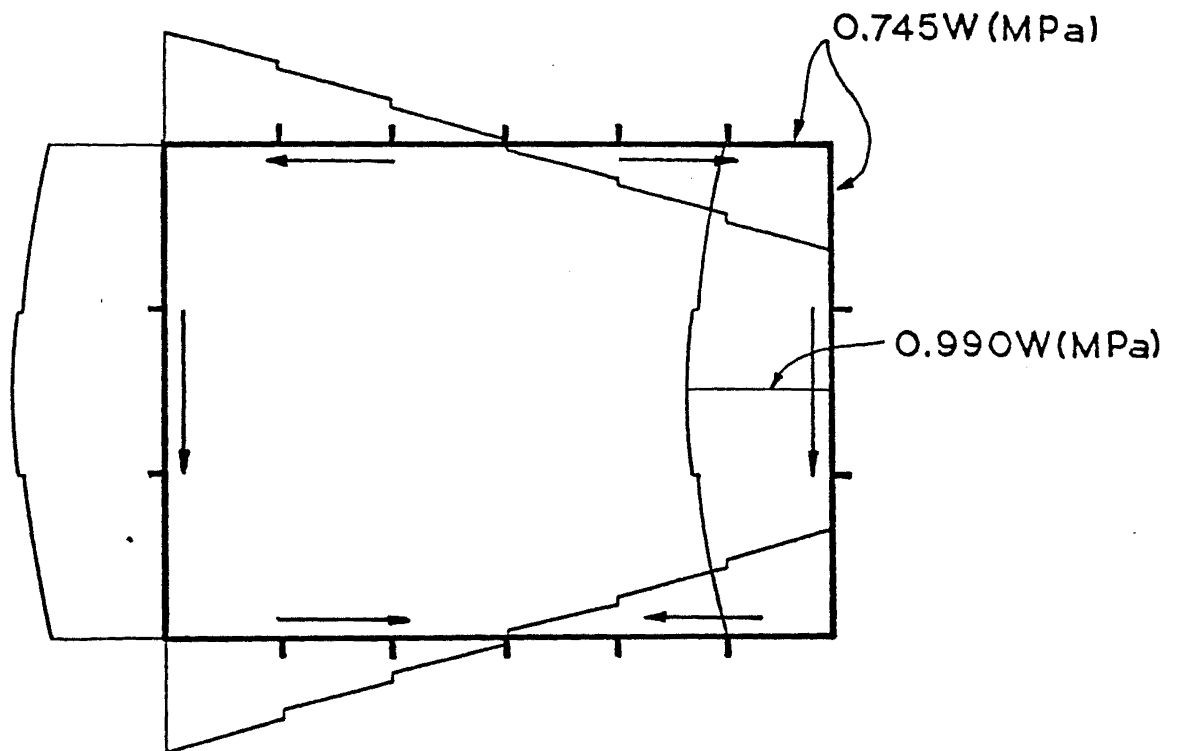


Fig. 6 Distribution of flexural shear stress  
in the cross section

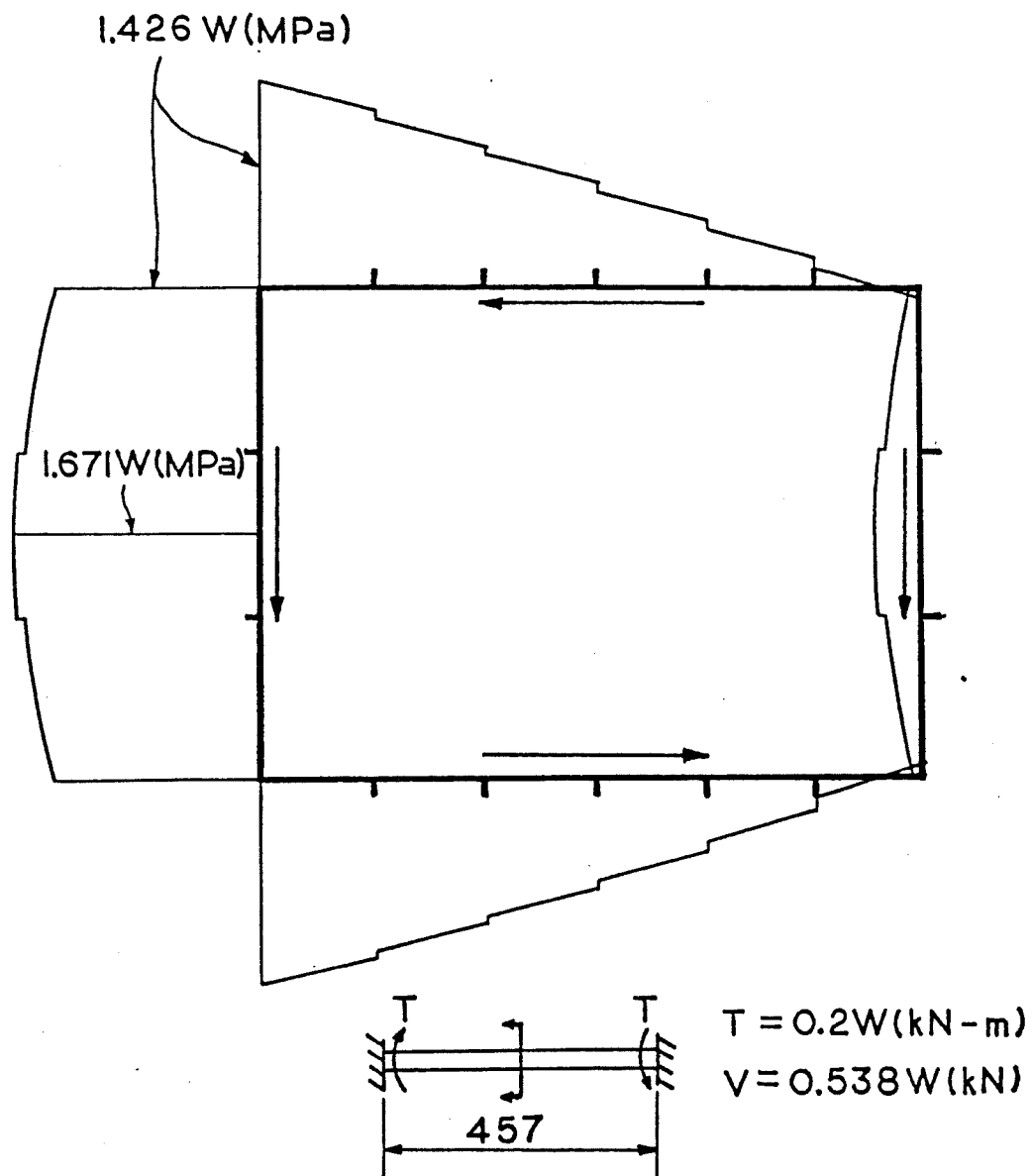


Fig. 7 Resultant shear stress in the cross section



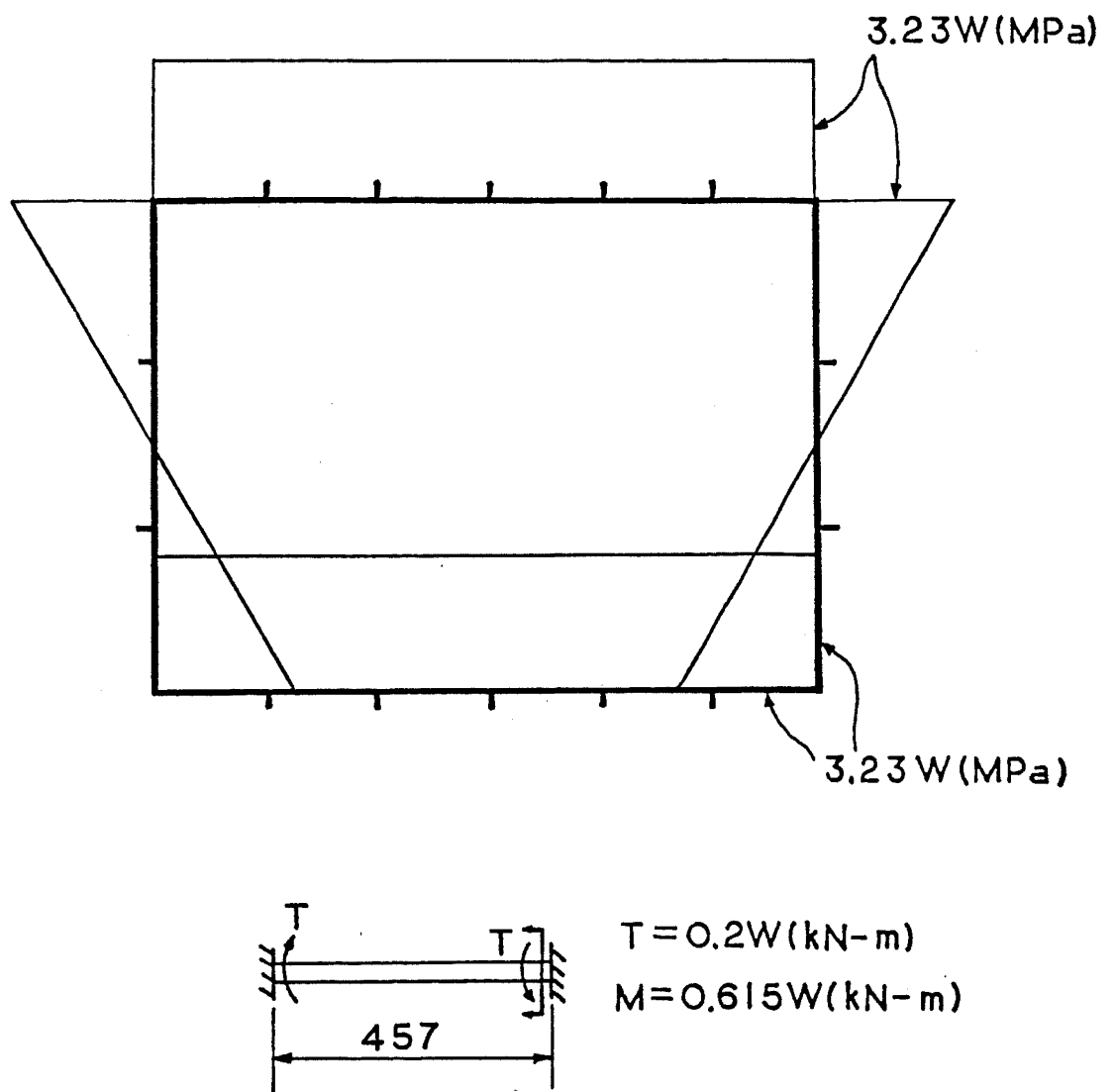


Fig. 8 Distribution of flexural normal stresses in the cross section according to simple beam theory

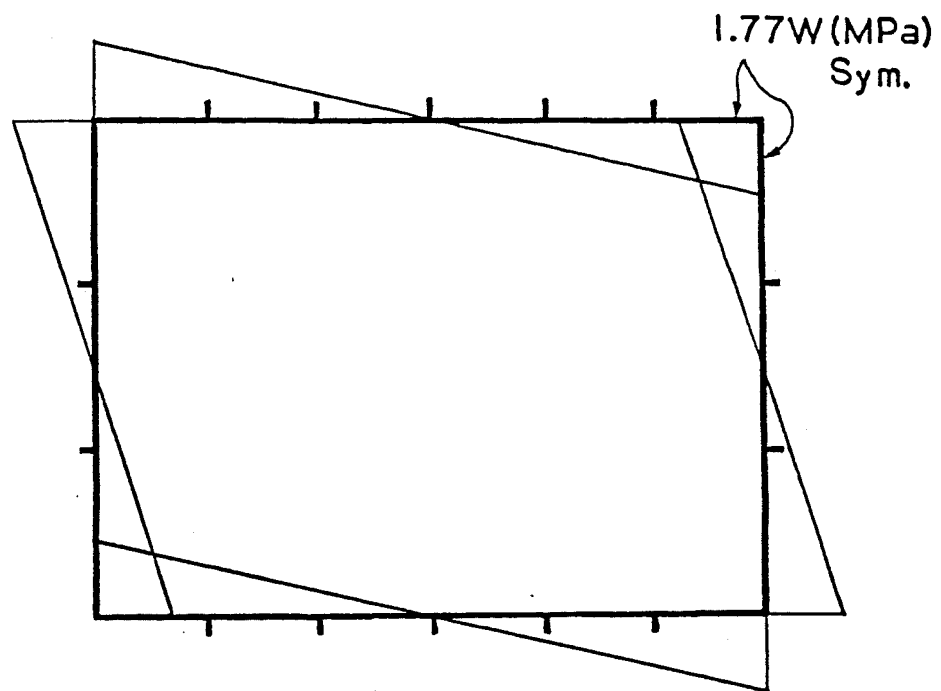


Fig. 9 Distribution of warping normal stress  
in the cross section

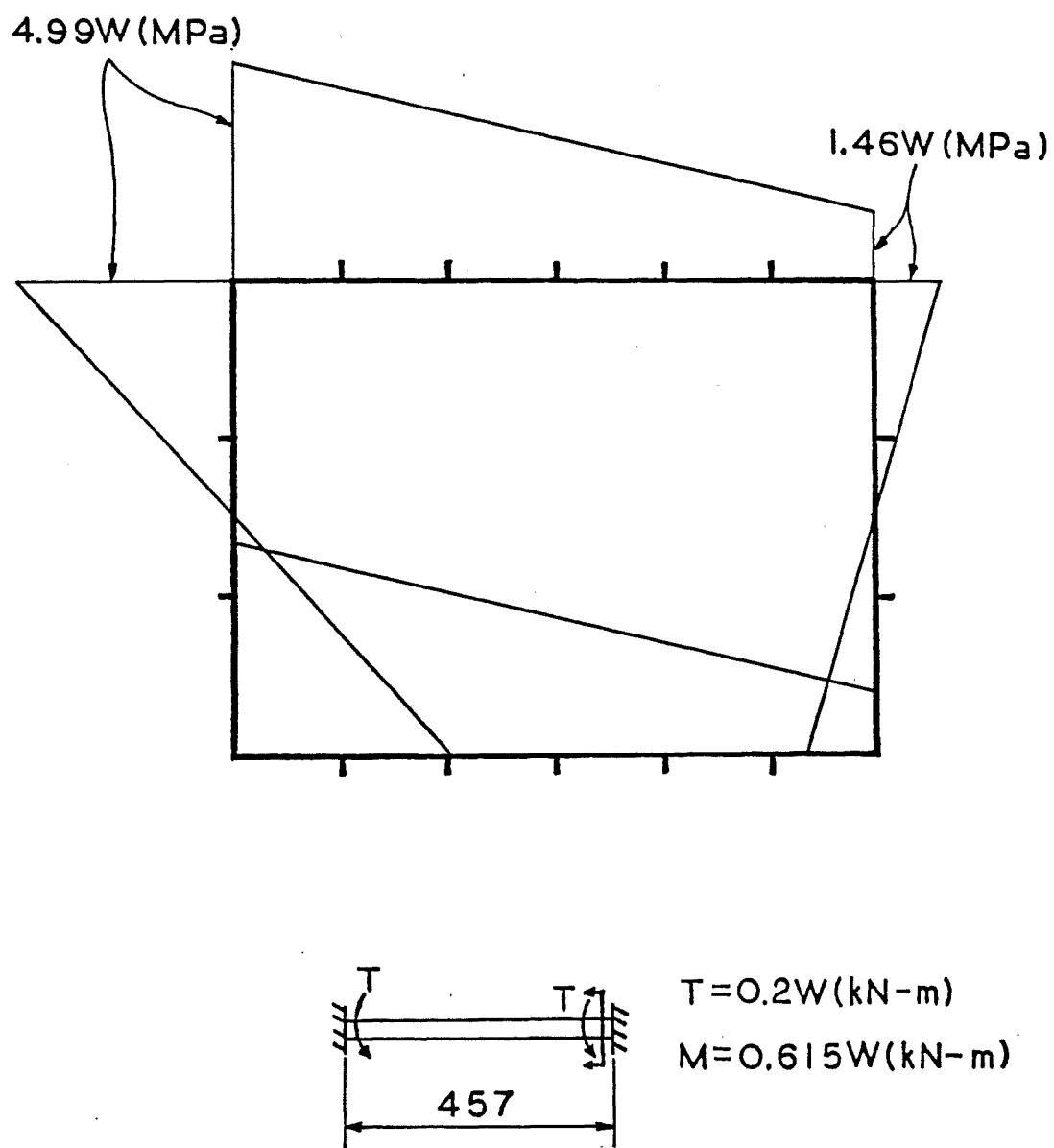


Fig. 10 Resultant normal stress in the cross section

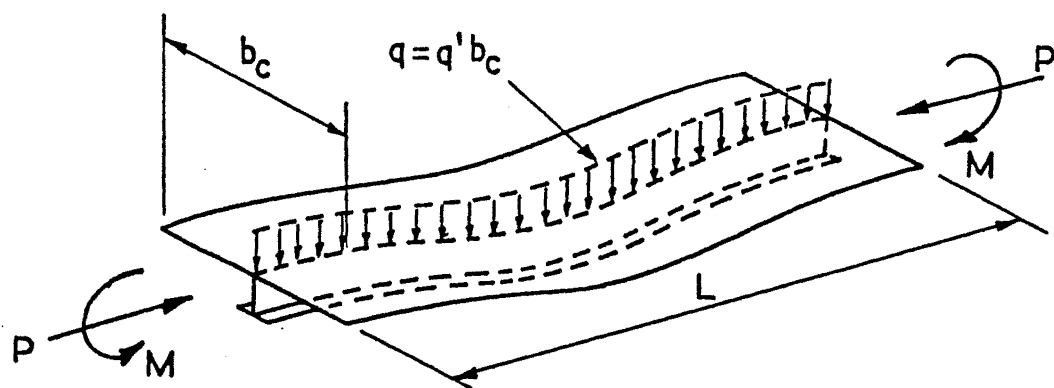


Fig. 11 Beam-column idealization

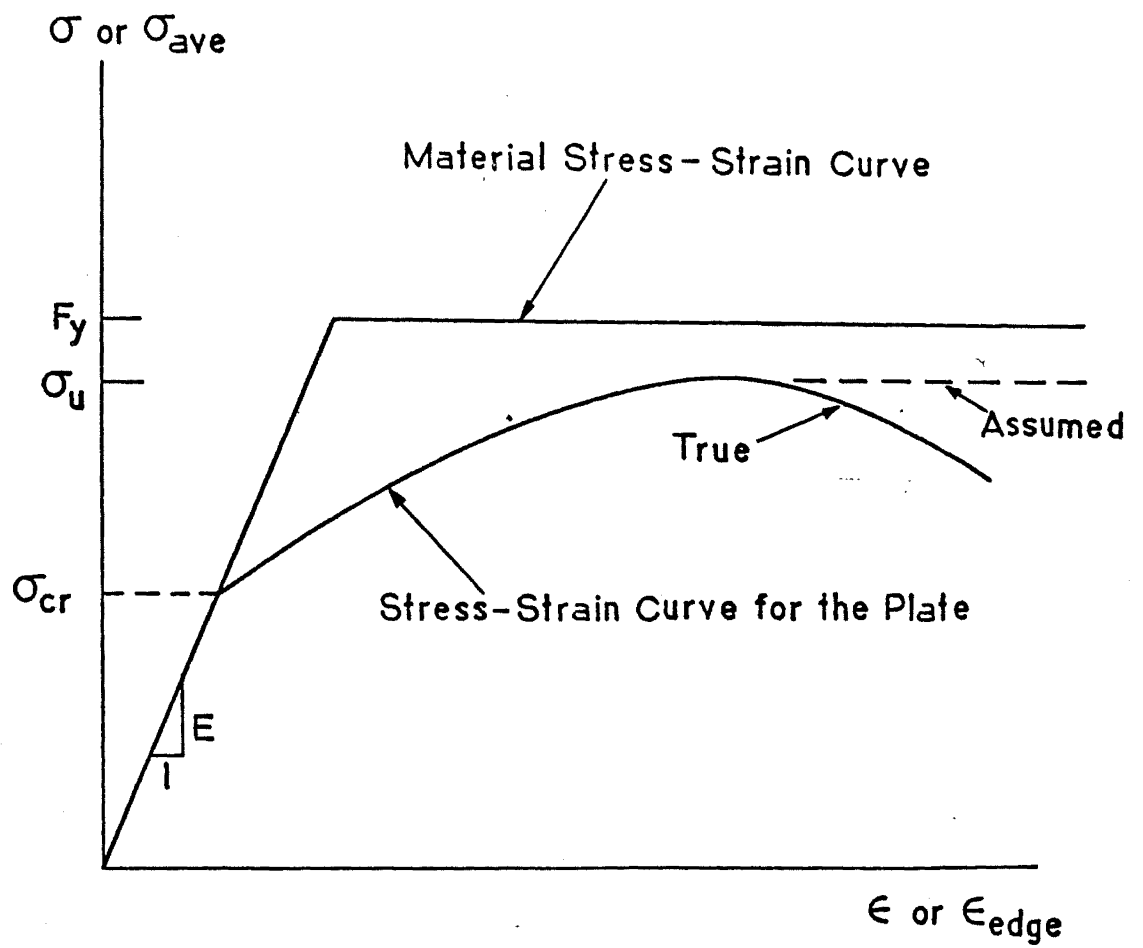
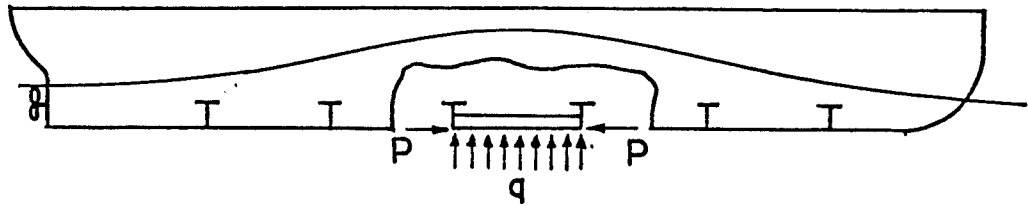
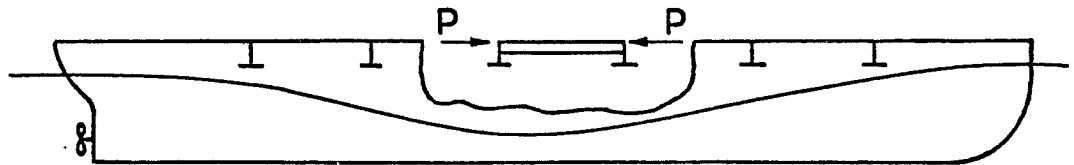


Fig. 12 Average stress vs. strain relationship  
for plate under compression



(A) HOGGING MOMENT

Fig. 13 Hogging Moment



(B) SAGGING MOMENT

Fig. 14 Sagging Moment

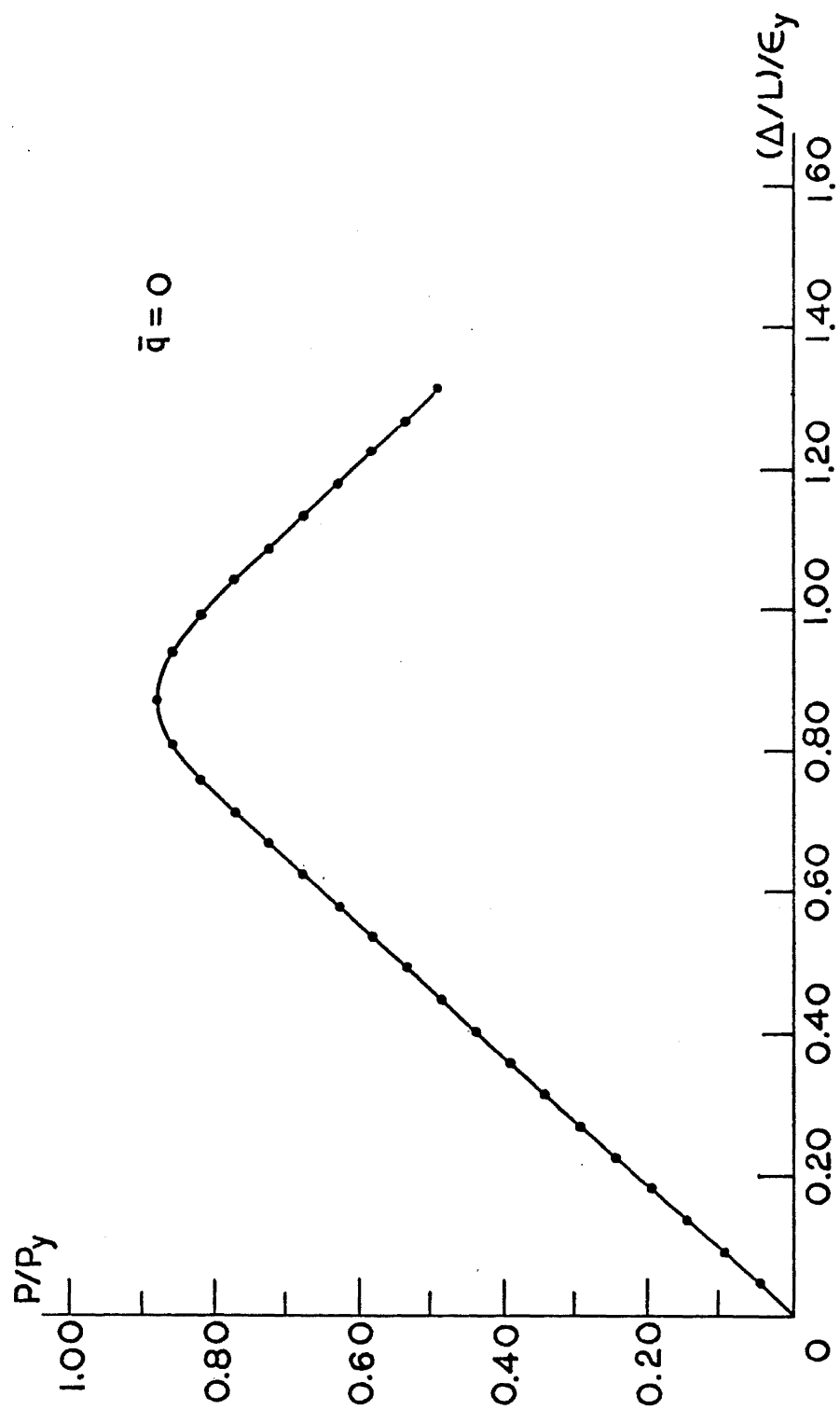


Fig. 15 Axial load vs. axial deformation relationship for  $\bar{q}=0$

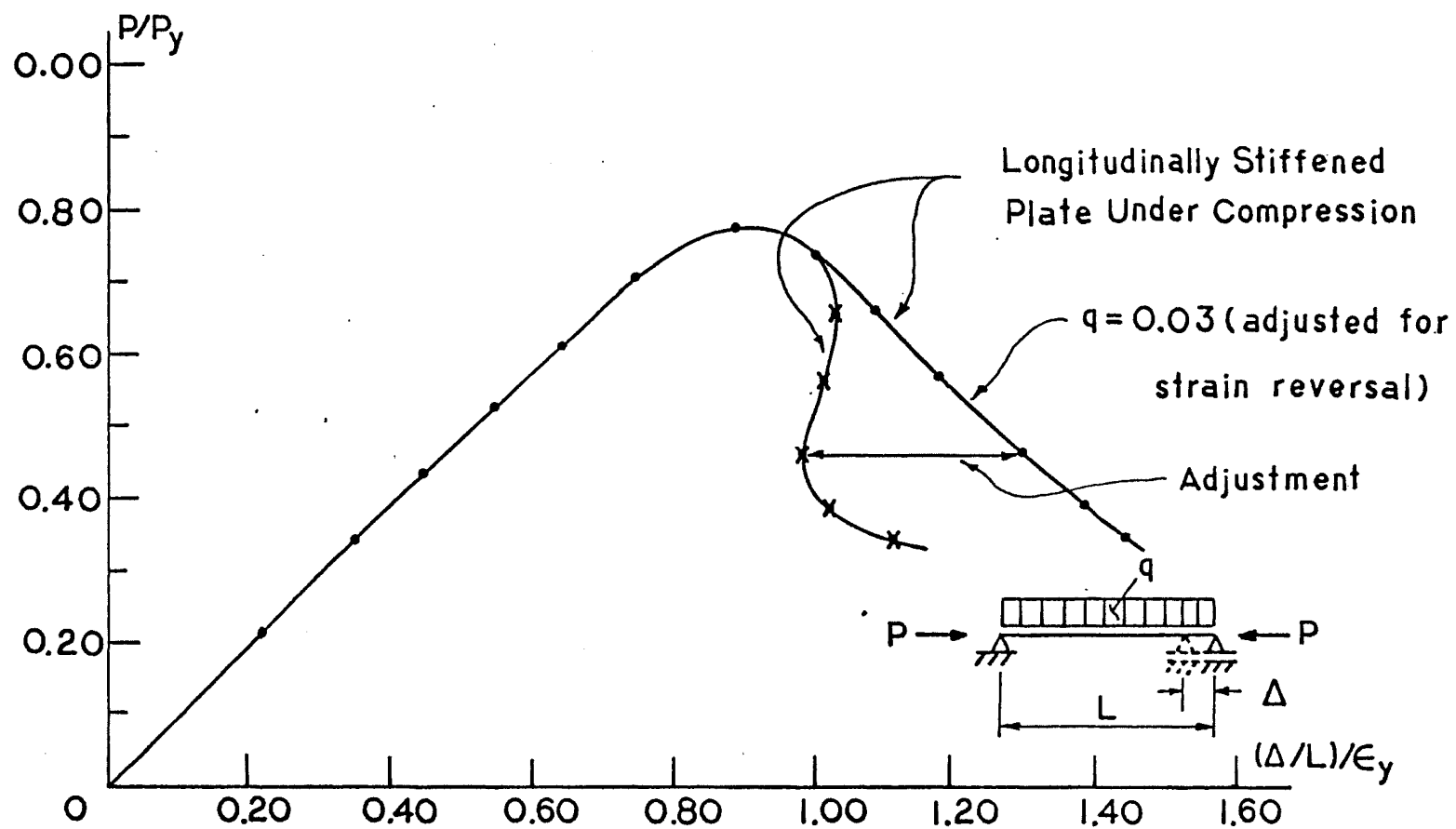
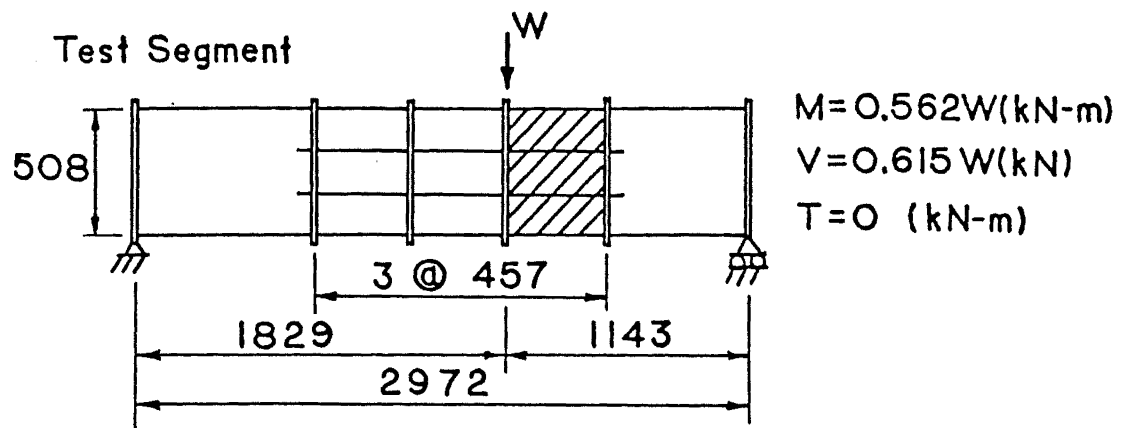
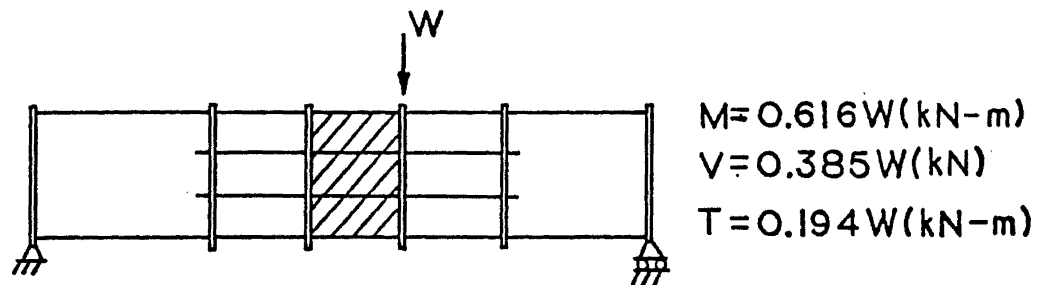


Fig. 16 Axial load vs. deformation relationship when strain reversal is not considered in post-ultimate range

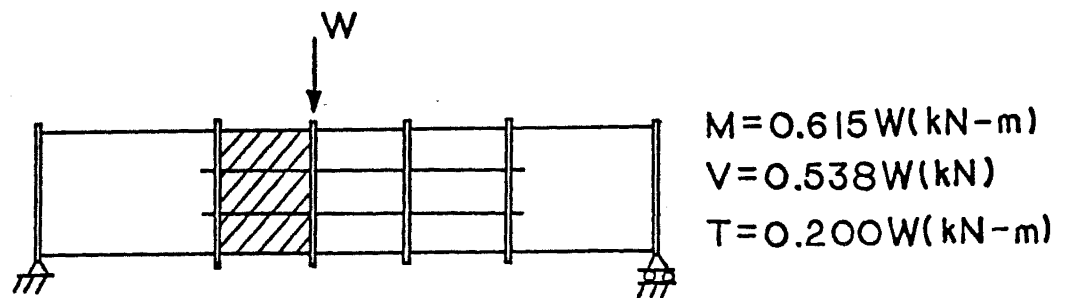




a) Test 1 : Moment + Shear



b) Test 2 : Moment + Shear + Torque



b) Test 3 : Moment + Shear + Torque

Fig. 17 Test Segments (all dimensions are in mm)

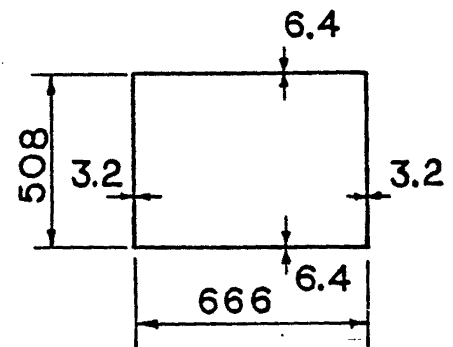
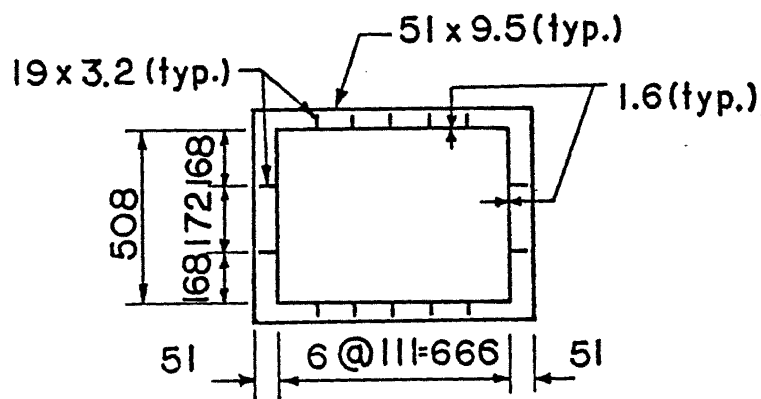
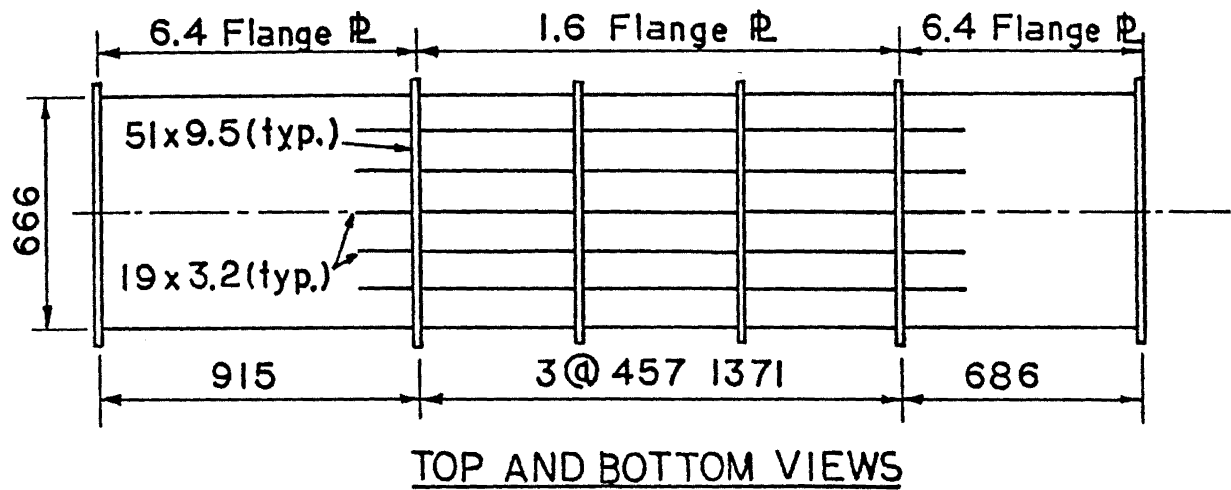
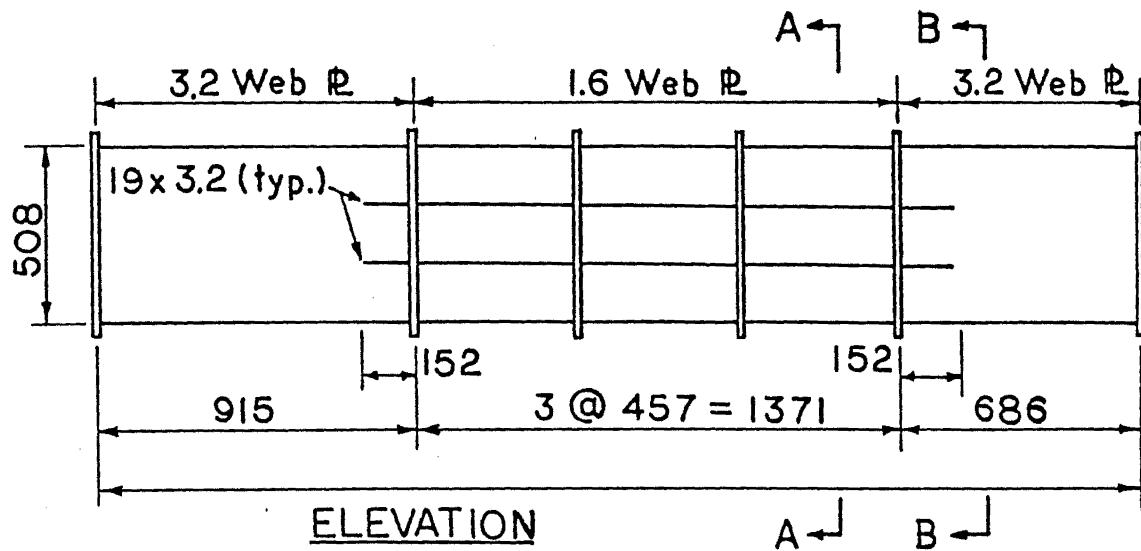


Fig. 18 Test Specimen Scantlings (all dimensions are in mm)

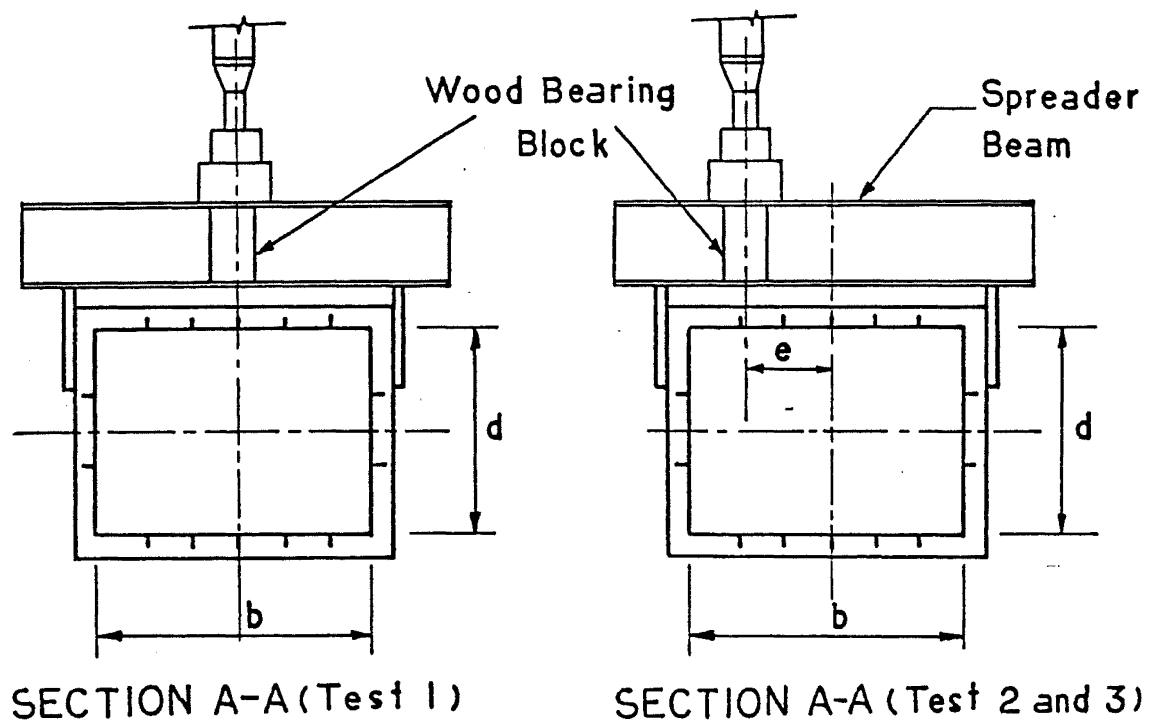
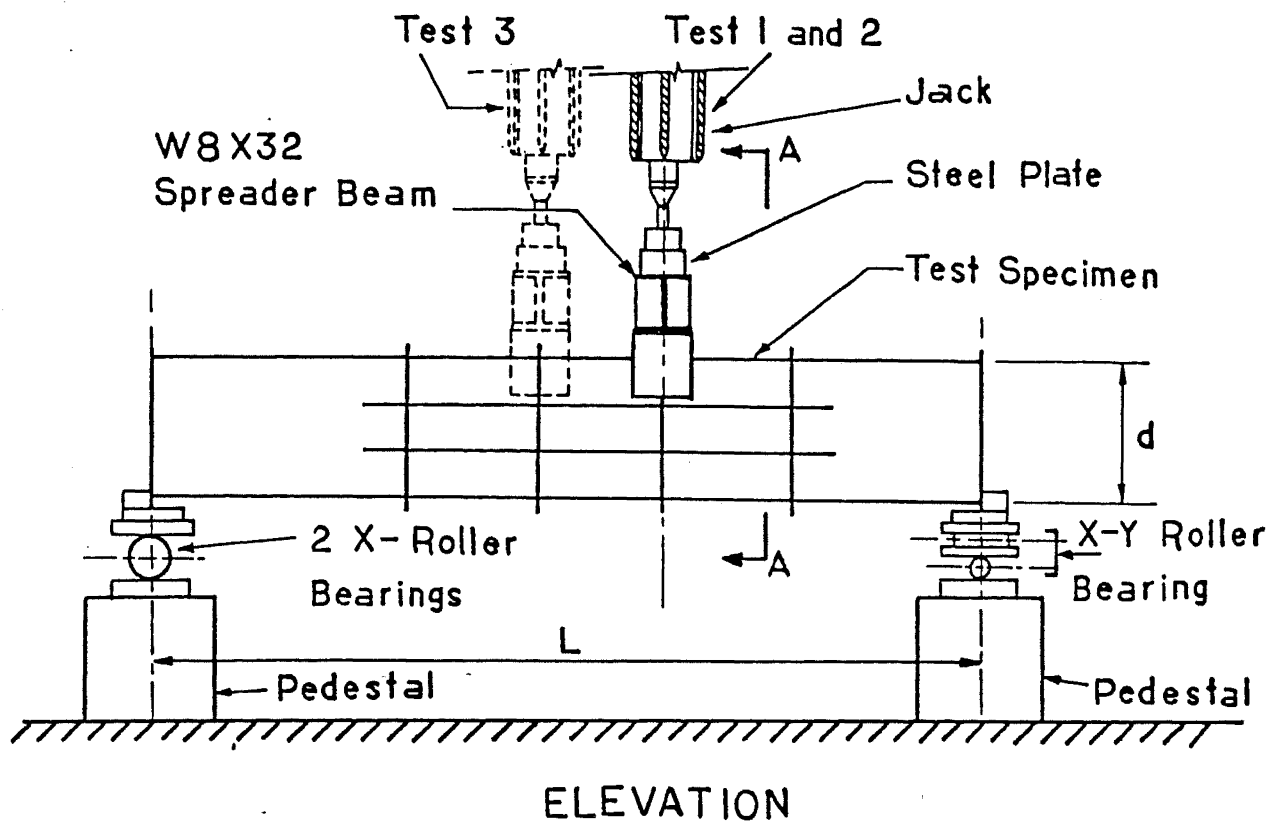


Fig. 19 Test Setup

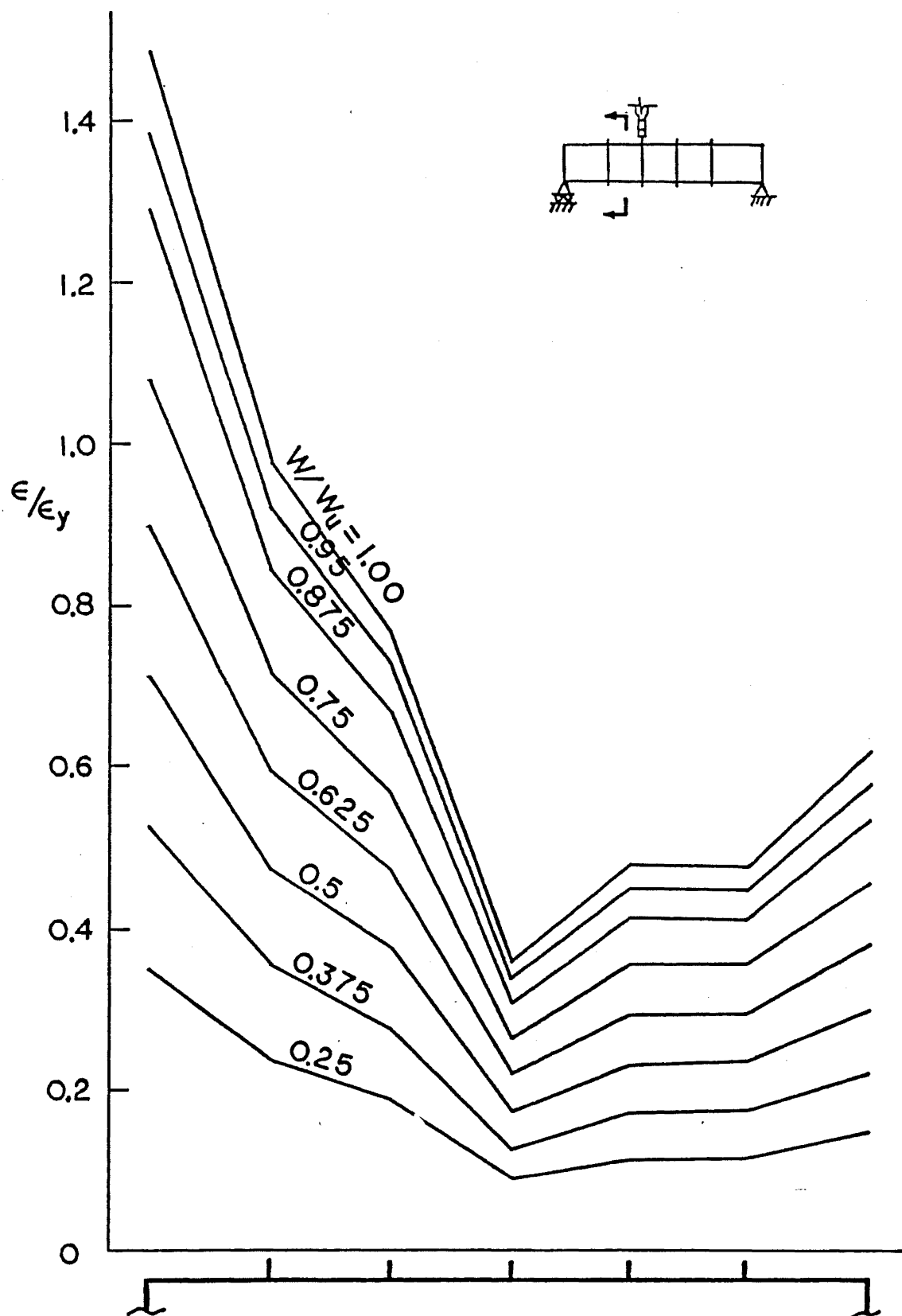


Fig. 20 Strain distribution across the compression flange at mid-length of Test segment(test 3)

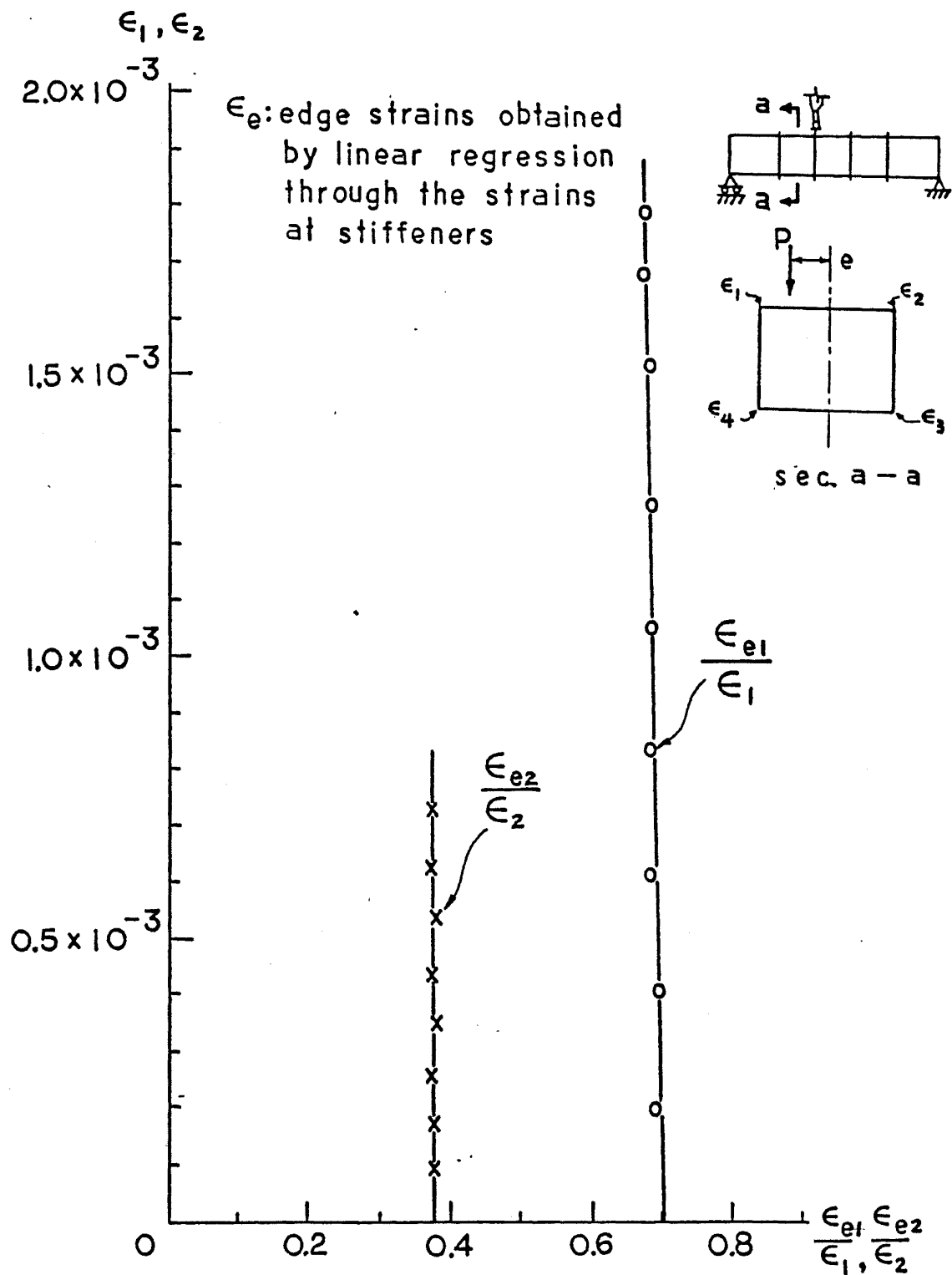


Fig. 21 Ratio of linearly fitted edge strains to observed edge strain readings for increasing load  $W$  as indicated by the value of  $\epsilon_1$  and  $\epsilon_2$